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APPLICATION OF FINITE ELEMENT MODELS OF AIRCRAFT STRUCTURES FOR AEROELASTIC FLUTTER ANALYSIS

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Abstract

The paper presents selected aspects of the application of finite element structural models (FEM) for numerical flutter analyses. The specificity of thin-walled airframe structures is taken into account, which means, that structures despite their relative lightness, must be suitably strong, stiff and elastically stable. The flutter phenomenon was briefly explained. Methods of solving the flutter problem are presented using analytical equations based on aerodynamic strip models (*Strip Theory*) or plain panel models with an assumption of local distributions of aerodynamic dipoles over aerodynamic elements (*Doublet Lattice Method*). Aspects of generating an element mesh based on adopted geometric models and the methodology of selecting material properties and mass discrete models for FEM are described. Selected structural models and application them in solving problems of natural vibrations and flutter are presented. Any thin-walled structure internally reinforced with longitudinal and transverse elements can be virtually recreated with the use of 2-dimensional elements (discs, plates, shells) and additional 1-dimensional elements (bars, beams). Models with specific mass and stiffness properties can then applied to solve detailed tasks in the field of statics, stability and dynamics of structures. Using the example of the presented light aircraft models, the modes of natural vibrations were determined. The computationally determined critical flutter speed will be as reliable as the mechanical properties of the numerical model are consistent with the real structure. In the process of adjusting properties of the structural models, there is the need to achieve compliance of numerical and experimental results. That means model displacements should be convergent with static test results as well as eigenfrequencies and eigenmodes should be comparable with results of ground resonance tests (GVTs).

Keywords: aeroelastic flutter, finite element method FEM, aircraft structural analysis

1. Specific qualities of thin-walled aircraft structures

Aircraft structures are subject to certain specific design regimes, which makes them lightweight and at the same time sufficiently stiff and durable in the context of possible deformations and stresses caused by external loads. There are at least four design postulates that the structure should meet in order to ensure the conditions of the desired level of durability and reliability of operation. They can be formulated in the form of design expectations characterized in the following points [1].

1) Ensuring the safety of the structure by appropriately selecting the ratio of ultimate loads (destructive loads) to the expected limit loads (permissible loads). The so-called safety factor recommended by the regulations for the construction of aircraft should be in the range of 1.5-2.0.
2) Ensuring the stress levels of the structure by selecting stresses that do not exceed the ultimate strength levels for characteristic directions.
3) Providing sufficient stiffness, for calculation purposes identified with the product values of the appropriate moduli of elasticity and specific cross-sectional quantities. For flexural stiffness, it is

the product of Young's modulus and the moment of inertia of the cross-section about the bending axis (*EJ*), while for torsional stiffness, it is the product of Young's modulus and the moment of inertia in torsion (GJ_s).

4) Providing lightness of the structure, where we mean the so-called relative lightness, resulting from the comparison of the weights of similar structural elements of different materials subjected to identical external loads. In these elements, due to their different thicknesses, different values of critical stresses appear, therefore the relative advantage of one material over another is determined by the ratios of density and maximum strength of these materials.

The specificity of thin structures allows for model simplifications in the context of numerical solutions using the finite element method (FEM). Any fragment can be approximated by a system of two-dimensional shell elements, and spatial models of airframe structures are most often developed as complex multi-assembly structures (fuselage, wings, stabilizers, rudders) built from two-dimensional finite elements, with possible supplementation with locally added sequences of one-dimensional elements. Avoiding the use of three-dimensional (solid) elements is most justified in this case. In this way, the number of model degrees of freedom (DOF) is significantly reduced and the computational structure is simplified by omitting the direction normal to the surface (z), which is irrelevant from the point of view of stresses. These simplifications, however, do not significantly affect the stiffness changes, but they accelerate the model creation process and shorten the calculation time.

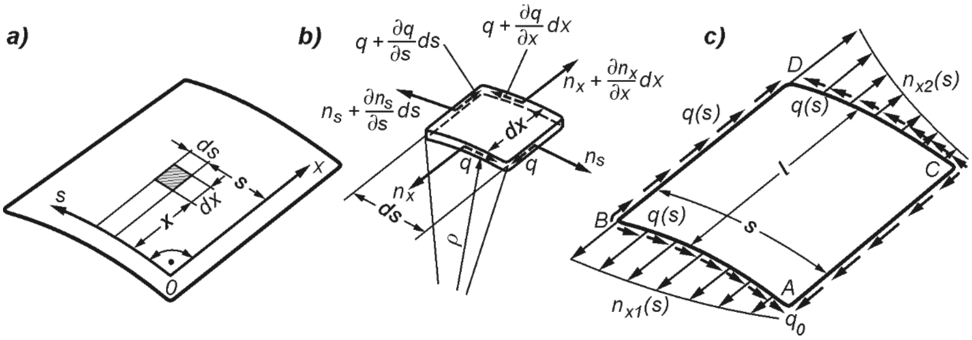


Figure 1 – A section of a thin-walled skin structure with a characteristic simplification of the interpretation of cross-sectional efforts – instead of normal and shear stresses (σ_x, σ_s, τ) stress intensities marked by n_x, n_s and $q(s)$ are used, respectively.

2. Formulation of the flutter problem and applicable methods of its solution

The analysis of aeroelasticity phenomena during the aircraft design process is one of the key computational stages implemented to ensure the durability and safety of the structure. Undesirable aeroelastic phenomena manifested by the deformability of the airframe structure result from the superposition of the effects of three different types of interactions. The simultaneously acting aerodynamic forces (A), elastic forces of the structure (E) and inertial mass forces (I) cause structural effects of static or dynamic nature. Aeroelasticity phenomena of aircraft structures are described in detail in classical literature [1], [2], [3]. The graphical representation of the effects is given in the form of a triangle of aeroelastic interactions, the so-called Collar triangle (Fig. 1).

The effects of aeroelastic interactions can be static or dynamic, as demonstrated for example in Fig. 2. In the case shown, the bending deformation of the swept wing leads to a change in the aerodynamic load distribution resulting in a forward displacement of the centre of pressure. This is therefore a destabilizing effect from the point of view of maintaining longitudinal stability. On the other hand, the deflection of the fuselage under the influence of its current weight load causes an increase in the angle of attack at the tail, resulting in an increase in the counterbalancing force on the tail. This effect therefore improves the longitudinal stability of the aircraft. The third possible effect is the bending deformation of the horizontal tail what could leads to a slight forward movement of the centre of pressure of the tail aerodynamic force.

Flutter is a phenomenon consisting in aerodynamically generated instability of structural vibrations during horizontal flight at constant speed. It manifests itself in the fact that above a certain speed called the critical flutter speed V_{kr}, vibrations with increasing amplitude appear [1, 2, 3, 4]. The reason

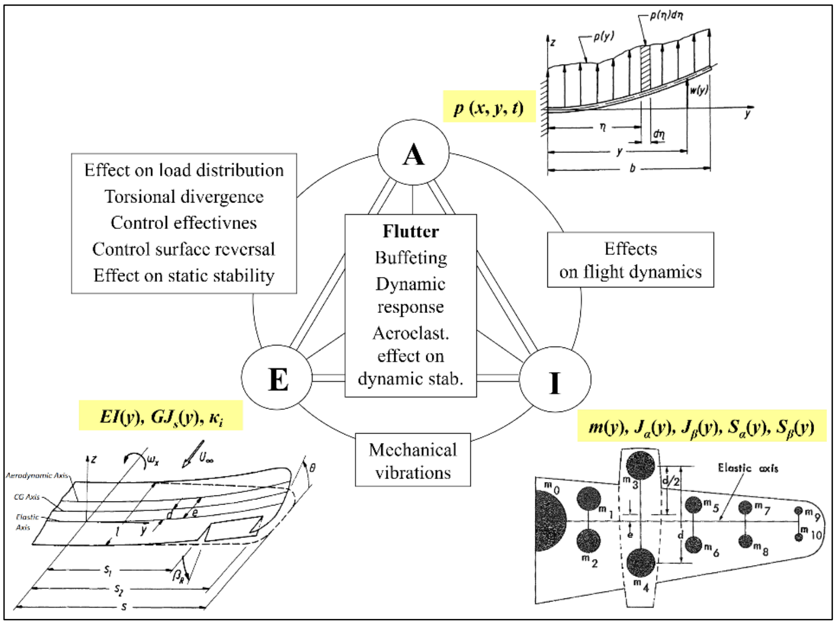


Figure 1 – Triangle of aeroelastic interactions and effects.

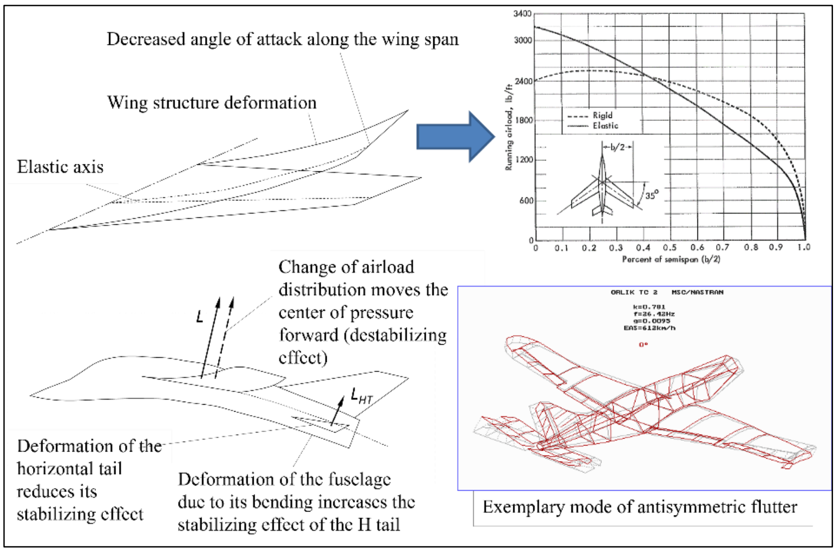


Figure 2 – Views of some exemplary static and dynamic aeroelastic effects.

for the phenomenon are unsteady aerodynamic forces generated as a result of elastic deformation of the airframe structure and, as a consequence, changes in the shape or position of the airfoil flowing through it. In addition to the unsteady aerodynamic forces Q(t), the aeroelastic object is affected by unsteady inertia forces $F_M(t)$, elastic forces of the structure $F_S(t)$ and damping forces $F_C(t)$ resulting from friction or viscous resistance. For a certain flight speed, the system of all unsteady forces can be described by the equation:

$$F_M(t) + F_C(t) + F_K(t) = Q(t) \tag{1}$$

which means that the non-stationary aerodynamic forces acting as external loads in the system under consideration are equal to the sum of the mass, damping and elasticity forces. Above a certain critical speed, equation (1) is no longer satisfied – the aerodynamic loads are so large that the sum of the remaining interactions conditioned by the mechanical properties of the structure does not balance the external forces. The forces Q(t) therefore drive the system to increasingly large oscillations, which consequently leads to damage or destruction of the structure.

The simplest example of an aeroelastic system for which flutter conditions can occur is an airfoil with three degrees of freedom defined as displacements: *h*, *α* and *β*. In the adopted notations, *h* is the translational displacement corresponding to the wing bending, while *α* and *β* are angular displacements corresponding to the torsional wing movement about its elastic and the deflection of the control surface (e.g. aileron). The system of equations describing the vibrations of an aeroelastic airfoil illustrated in Fig. 3 with exciting generalized forces defined for three degrees of freedom Q_h, Q_α and Q_β has the following form:

$$\begin{cases} M\ddot{h} + S_\alpha\ddot{\alpha} + S_\beta\ddot{\beta} + \frac{M\omega_h^2 g_h}{\omega}\dot{h} + M\omega_h^2 h = Q_h \\ S_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + \left[(c-a)bS_\beta + I_\beta\right]\ddot{\beta} + \frac{I_\alpha\omega_\alpha^2 g_\alpha}{\omega} + I_\alpha\omega_\alpha^2\alpha = Q_\alpha \\ S_\beta\ddot{h} + \left[(c-a)bS_\beta + I_\beta\right]\ddot{\alpha} + I_\beta\ddot{\beta} + \frac{I_\beta\omega_\beta^2 g_\beta}{\omega}\dot{\beta} + I_\beta\omega_\beta^2\beta = Q_\beta \end{cases} \tag{2}$$

where characteristic the circular eigenfrequencies corresponding to the basic unconjugate eigenmodes are the following:

$$\omega_h = \sqrt{\frac{K_h}{M}} \qquad \omega_\alpha = \sqrt{\frac{K_\alpha}{I_\alpha}} \qquad \omega_\beta = \sqrt{\frac{K_\beta}{I_\beta}} \tag{3}$$

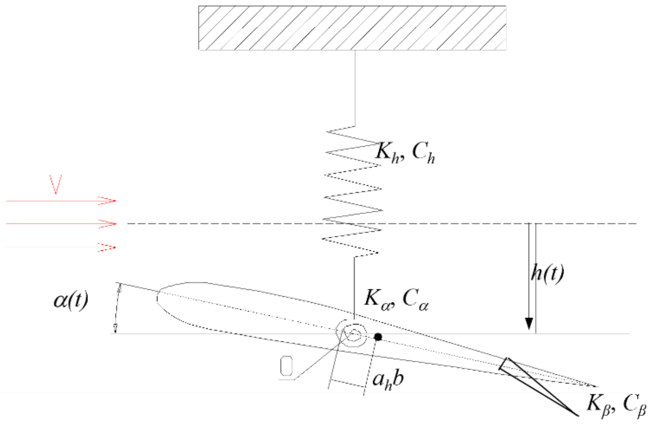


Figure 3 – Aeroelastic airfoil with three degrees of freedom subject to fluttering.

The key symbols applied in above terms refer to mass or inertia qualities (M, I_α, I_β) and to stiffness value suitable for specific degrees of freedom (K_h, K_α, K_β). The aerodynamic force coefficients Q_i (where i=h, α, β)are complex functions whose complex values depend on the characteristic geometric dimensions of an airfoil, the air density and two dimensionless aerodynamic numbers: the Mach number $Ma=V/a$ and the Strouhal number $k=\omega b/V$. The latter is interchangeably called reduced frequency. Equations (2) are reduced to a homogeneous form:

$$\begin{cases} A'\frac{h}{b} + B'\alpha + C'\beta = 0 \\ D'\frac{h}{b} + E'\alpha + F'\beta = 0 \\ G'\frac{h}{b} + H'\alpha + I'\beta = 0 \end{cases} \tag{4}$$

and then solved by the determinant method with respect to the unknown *X* involved in the coefficients *A'*, *E' I'* on the main diagonal of the characteristic matrix:

$$\begin{vmatrix} A' & B' & C' \\ D' & E' & F' \\ G' & H' & I' \end{vmatrix} = 0 \tag{5}$$

For the determined complex solutions X_{cr}, the values of the critical parameters are then determined – the frequency of coupled vibrations and the critical flutter velocity. Both parameters are dependent on reduced frequency k, and presents in the following way:

$$\omega_{cr} = \sqrt{\frac{1}{X_{cr}}} \qquad V_{cr} = b\omega_{cr}\left(\frac{1}{k}\right)_{cr} = b\sqrt{\frac{1}{X_{cr}}}\left(\frac{1}{k}\right)_{cr} \tag{6}$$

A more advanced stage of development of the strip method in application to calculations of flutter not only of an oscillatory airfoil or a lifting surface, but of the entire aircraft is the JG2 software developed in the 1980s and 1990s in the Polish Aviation Works (PZL Mielec) [9, 10]. The program solves the problem of flutter with the participation of strip aerodynamics, but the solution is obtained globally for the entire aircraft. The APRMH and APRSC programs approximate the distributions of mass and stiffness parameters with respect to the axes characteristic of structural units. The matrix equation of flutter is then solved in the FLATH program, whereby the data for the program include aerodynamic forces for the distribution of local strips along the length of the wings and tail surfaces. An example of the program's application to calculate flutter of the PZL-130 Orlik aircraft is demonstrated in Fig. 4.

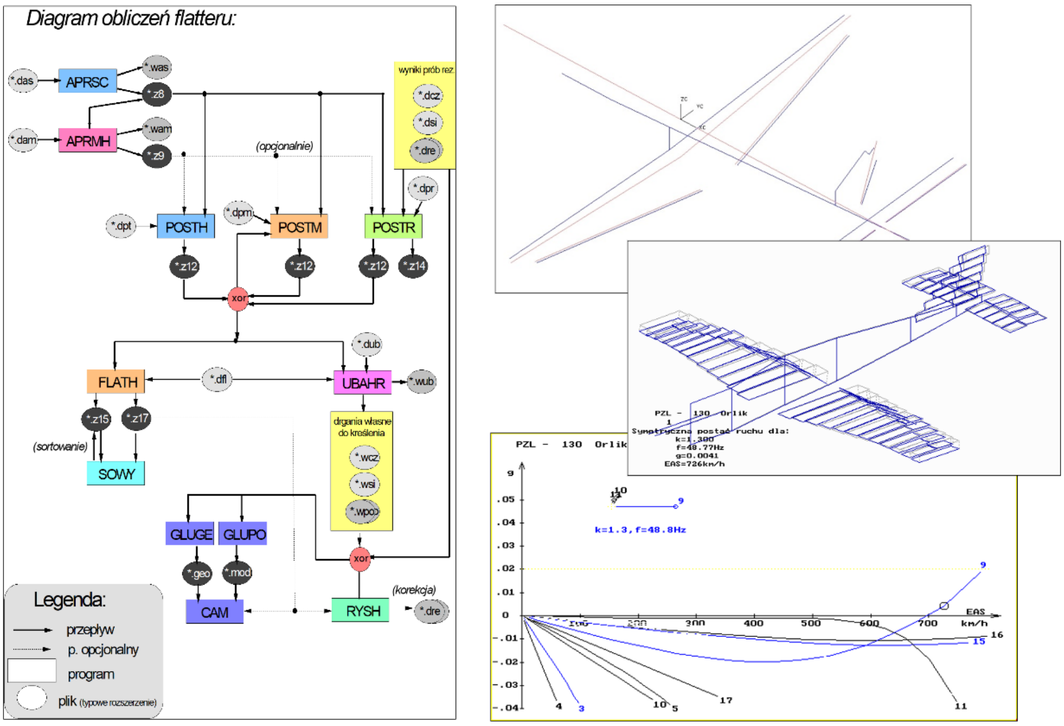


Figure 4 – Data flow diagram in JG2 Software applicable for normal modes and flutter calculations [9, 10], and the exemplary case – symmetric flutter of PZL-130 turboprop trainer.

In Nastran, the flutter solution (SOL 145) can be implemented using one of three available calculation methods called K, KE and PK methods, which differ in the way of declaring the damping matrix [4, 5]. The PK method used in the own calculations solves the flutter equation formulated as a complex eigenproblem:

$$\left[M_{hh}p^2+\left(B_{hh}-\frac{1}{4}\frac{\rho b V Q^I_{hh}}{k}\right)p+\left(K_{hh}-\frac{1}{2}\rho V^2 Q^R_{hh}\right)\right]\{u_h\}=0 \qquad (7)$$

where the individual terms denote respectively:

$Q^I_{hh}(Ma,k)$ – imaginary part of the complex aerodynamic matrix, the so-called modal aerodynamic damping matrix, a function of the Mach number Ma and the reduced frequency $k=\omega b/2V$,,

$Q^R_{hh}(Ma,k)$ – real part of the complex aerodynamic matrix, the so-called modal aerodynamic stiffness matrix, a function of the parameters Ma and k,

$p=\omega(\gamma\pm i)$ – complex eigenvalue, where ω is the vibration frequency and γ is the transition damping rate coefficient (then the structural damping $g=2\gamma$).

The *p* value is determined from the equation for the user-defined range of parameters *Ma*, *V* and *ρ*. In the iterative calculation mode, the numbers *ω* and *k* are selected, related to each other by the relation defining the reduced frequency. Finally, after solving equation (7), the complex eigenvalues *p* are obtained. From the real and imaginary parts of the number *p*, the vibration frequencies *ω* and the structural damping *g* are determined according to the following conversion relations:

$$\omega = Im(p), \quad g = 2\gamma = \frac{2\cdot Re(p)}{Im(p)} \qquad (8)$$

The frequency and damping curves as a function of velocity V are then plotted on graphs as the so-called flutter curves *g*(*V*) and *f*(*V*). The intersection point of the curve *g*(*V*) with the horizontal axis g=0 is the so-called critical point, and the velocity corresponding to this point is the critical flutter velocity.

Flutter calculations are performed in Nastran based on integrated aeroelastic models created by linking the structural model with the panel aerodynamic model. Based on the FEM model, the modal mass and stiffness matrices are determined, i.e. $[M_{hh}]$ and $[K_{hh}]$, while the panel element and slender element model is designed to determine the components of the complex aerodynamic matrix $\left[Q_{hh}^{R} + iQ_{hh}^{I}\right]$. The values of the force coefficients in the so-called aerodynamic nodes are calculated based on formulas defined for unsteady aerodynamics based on the *Doublet Lattice Method* (*DLM*) – Fig. 5 and *Slender Body Theory* (*SBT*) methods. In this case, the displacements of aerodynamic elements are interpolated with imposed spline functions according to the displacements of structural nodes [4, 5, 6, 7].

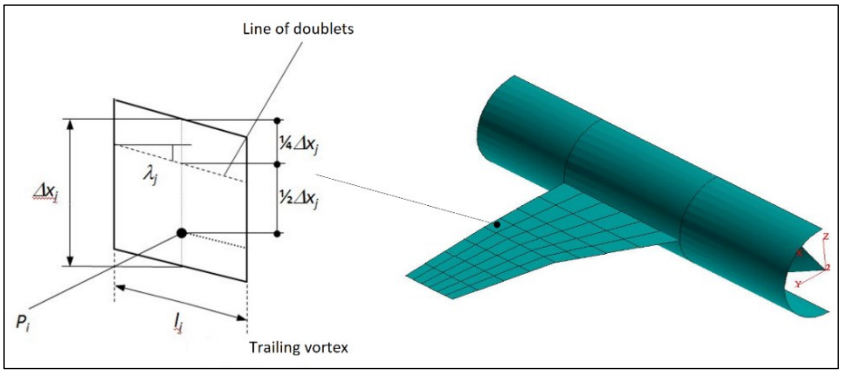


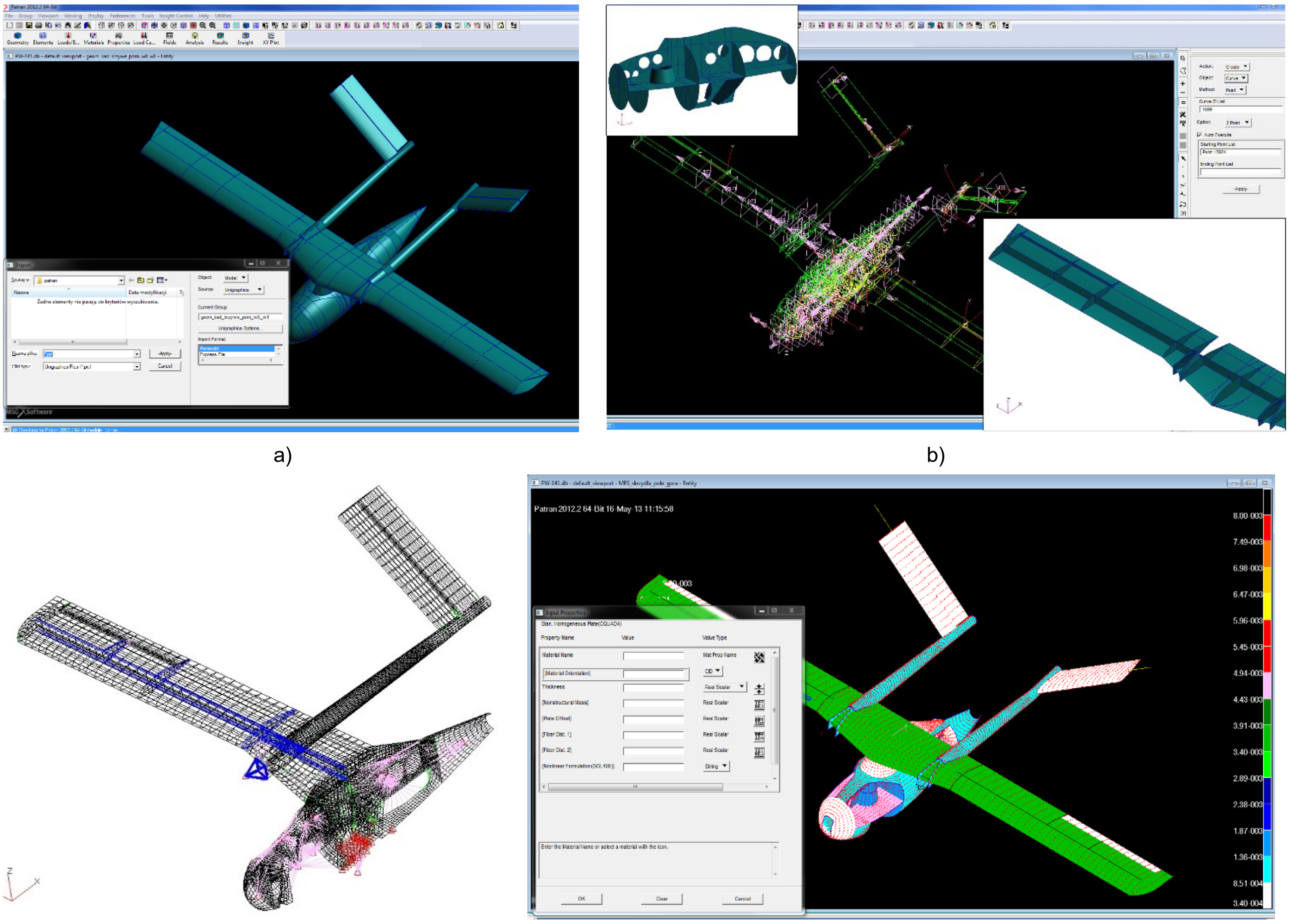
Figure 5 – Aerodynamic model for non-stationary aerodynamics including panel for DLM method and slender body for SBT.

3. Developing structural models for FEM

Aircraft structure modelling leading to the development of a model simulating mass and stiffness properties is a very laborious and long-term process [2]. In a comprehensive cycle of aircraft model generation using advanced CAD/CAE environments (Siemens NX, Patran), characteristic stages of activities can be distinguished:

- design/measurement of the external geometry of the airframe,
- development of a geometry model in the CAD environment,
- discretization of the virtual structure while maintaining the thin-walled nature of the structure,
- identification of construction materials (material constants: E, G, ρ, ν),
- assigning physical properties to model mesh elements (materials, thicknesses, masses),
- localization of concentrated masses imitating on-board equipment elements and loads,
- definition of fixed connections (fittings) - e.g. wing-fuselage, stabilizer-fuselage,
- introduction of movable hinge connections for control surfaces with surrogate stiffness of controls,
- simulation of model support corresponding to real boundary conditions.

As can be seen, in order to recreate the actual structural properties, it is necessary to identify the geometry of the object as well as to know its structural, mass and material parameters. The starting point for starting the model structuring of the airframe is the geometry model of the aircraft. In a situation where a model of an aircraft in service is to be developed, for which the design geometry is not available, such a model should be generated using measurement techniques and geometry reproduction methods characteristic of reverse engineering processes (RE) and CAD systems. An example process of developing the FEM model is demonstrated in Fig. 6.



a) b)

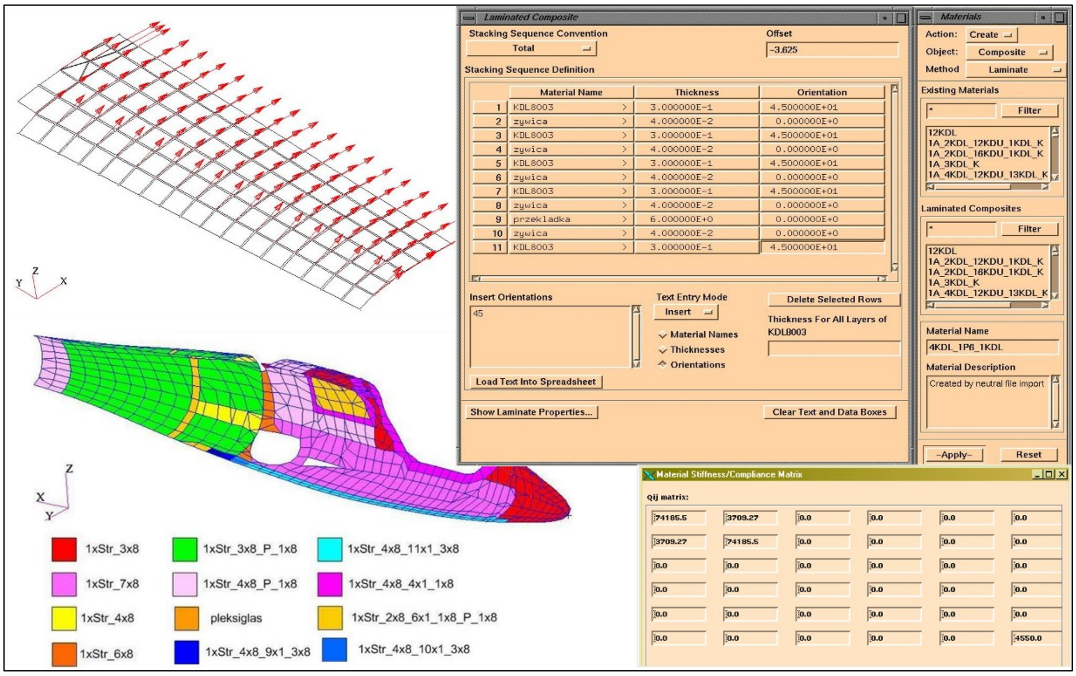
c) d)

Figure 6 – Development stages of the structural model developed in the FEM preprocessor – case of developing FEM model of UAV Samonit:
a) geometric model of the aerodynamic body, b) detailing of the geometry models of structural units, c) discrete element model, c) structural model with assigned material properties.

Before developing a discrete model, it is necessary to carry out work and measurements that allow to identify actual design parameters. These parameters can be grouped into the following types:

- design-configuration – characterize the type, arrangement and number of used components or elements (aspect ratio, convergence, wing and tail sweep, number of spars, walls, frames, ribs, stringers and their location);
- main geometric dimensions;
- material – physical constants of used structural materials (stiffness moduli, Poisson's ratios, densities) – modeling of the composite structure based on multilayer material models is shown in Fig. 7;
- mass – masses and mass moments of inertia of structural units, aggregates, devices, on-board cargo, suspended elements (pods);
- kinematic and rigid – directions of the axes of the hinged connections of the rudders, ranges of rudder deflections, stiffness of control systems;
- technological – e.g. thickness of elements, matching of contact surfaces, types of connections.

Parameters related to dimensions and structural materials affect the global stiffness matrix **K**, while the distribution and mass values determine the global inertia matrix **M** of the model. In case of aircraft in service, data is obtained from direct measurements or from documentary reconnaissance. Assessing dimensions and structural parameters is very difficult providing that only operational documentation is available.



Stacking Sequence Convention: Total

Offset: -3.625

Stacking Sequence Definition

	Material Name	Thickness	Orientation
1	KDL8003	3.000000E-1	4.500000E+01
2	zywica	4.000000E-2	0.000000E+0
3	KDL8003	3.000000E-1	4.500000E+01
4	zywica	4.000000E-2	0.000000E+0
5	KDL8003	3.000000E-1	4.500000E+01
6	zywica	4.000000E-2	0.000000E+0
7	KDL8003	3.000000E-1	4.500000E+01
8	zywica	4.000000E-2	0.000000E+0
9	przekladka	6.000000E+0	0.000000E+0
10	zywica	4.000000E-2	0.000000E+0
11	KDL8003	3.000000E-1	4.500000E+01

Insert Orientations: 45

Text Entry Mode: Insert

◇ Material Names
◇ Thicknesses
◆ Orientations

Thickness For All Layers of KDL8003

Action: Create
Object: Composite
Method: Laminate

Existing Materials: *

12KDL
1A_2KDL_12KDU_1KDL_K
1A_2KDL_16KDU_1KDL_K
1A_3KDL_K
1A_4KDL_12KDU_13KDL_K

Laminated Composites: *

12KDL
1A_2KDL_12KDU_1KDL_K
1A_2KDL_16KDU_1KDL_K
1A_3KDL_K
1A_4KDL_12KDU_13KDL_K

Material Name: 4KDL_1P6_1KDL

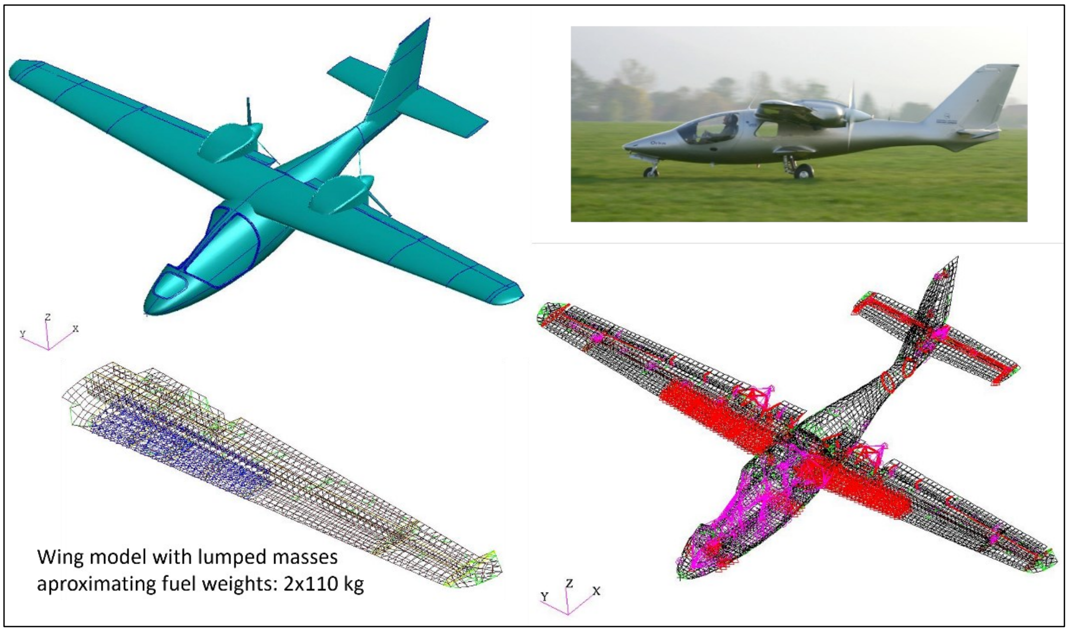
Material Description: Created by neutral file import

Qij matrix:

74185.5	3709.27	0.0	0.0	0.0	0.0
3709.27	74185.5	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	4550.0

Figure 6 – Developing composite structures based on multilayer assembly models compound of stacked fabric plyes (2D-orthotropic layers).

Further analyses were carried out for models of light General Aviation class aircraft. Figures 8 and 9 show the models of the EM-11 Orka and MUT Osa airplanes. For both the cases structural FE models were developed whith integrated with multi-panel aerodynamic models forn non-stationary aerodynamics and then used for numerical flutter calculations carried out in MD Nastran Software.



Wing model with lumped masses aproximating fuel weights: 2x110 kg

Figure 7 – Geometry and FE model of EM-11 airplane.

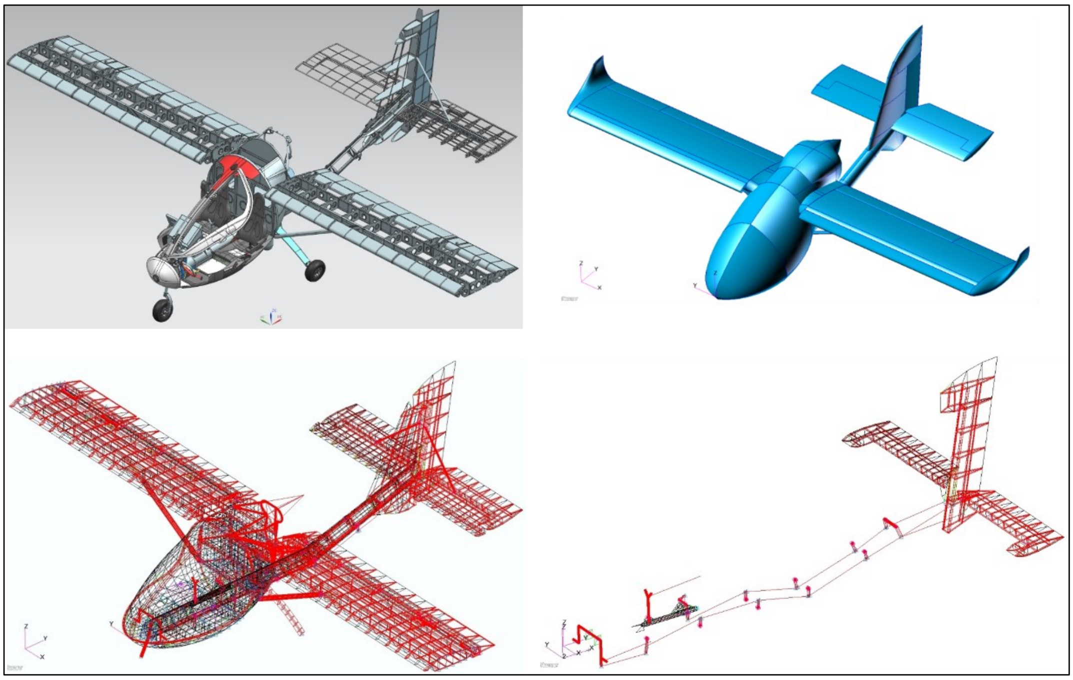


Figure 8 – Geometry and FE model of MUT Osa airplane (structure with elastic control elements).

4. Selected flutter cases for light aircraft

With a complete mass-stiffness model, it was possible to perform numerical analysis to determine the form and frequency of natural vibrations. The solution to such a problem consists in finding the eigenvalues of the matrix equation describing the natural undamped vibrations of the model. The basic forms of natural vibrations obtained for the basic versions of both aircraft are shown in Figures 9 and 10.

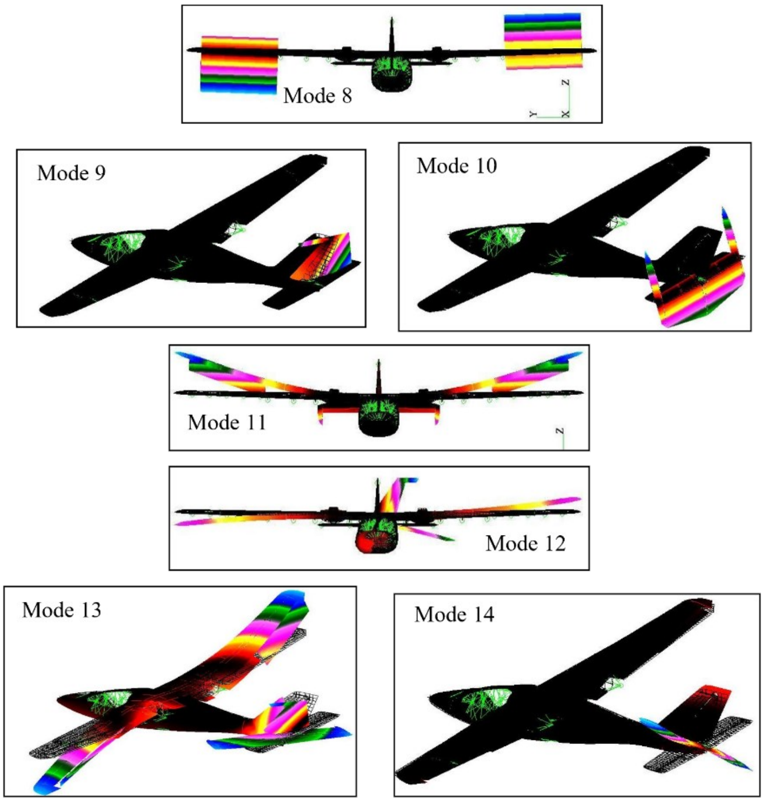


Figure 9 – Basic normal modes of EM-11 Orka aircraft.

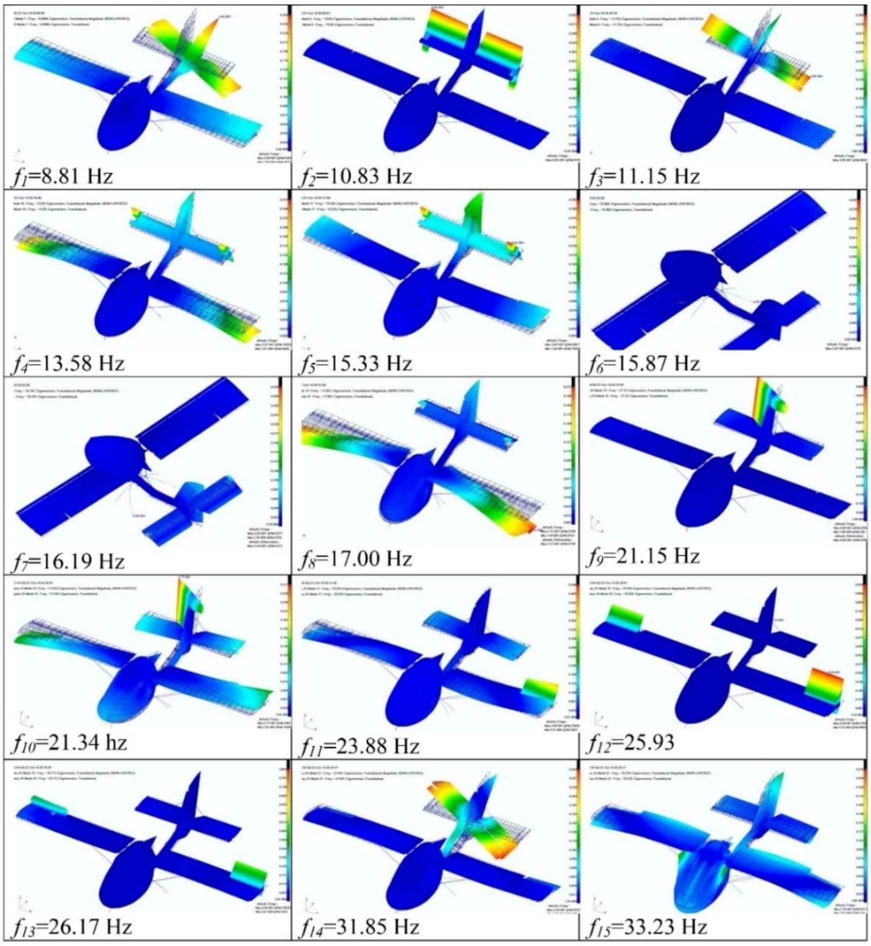


Figure 10 – Basic normal modes of Osa aircraft.

4.1. Flutter results for EM-11 airplane

Two characteristic configurations were taken into consideration in analysis. Case 1 was analysed for the model with free controls, case 2 was analysed for the model with controls-like elastic elements.

Case 1 – MTOW (1650 kg), control balancing masses consistent with the prototypes, free control, first 15 deformable modes included – symmetrical flutter of the horizontal tail with the participation of the H stabilizer bending, vertical fuselage bending and H rudder deflection (V_{kr}=378 km/h and f_f=13.1 Hz).

Case 2 – MTOW (1650 kg), control balancing masses consistent with the prototypes, control stiffened with control system elements, first 16 deformable modes included – antisymmetrical flexural-aileron flutter and vertical tail control flutter (V_{kr}=378 km/h and f_f=10.3 Hz).

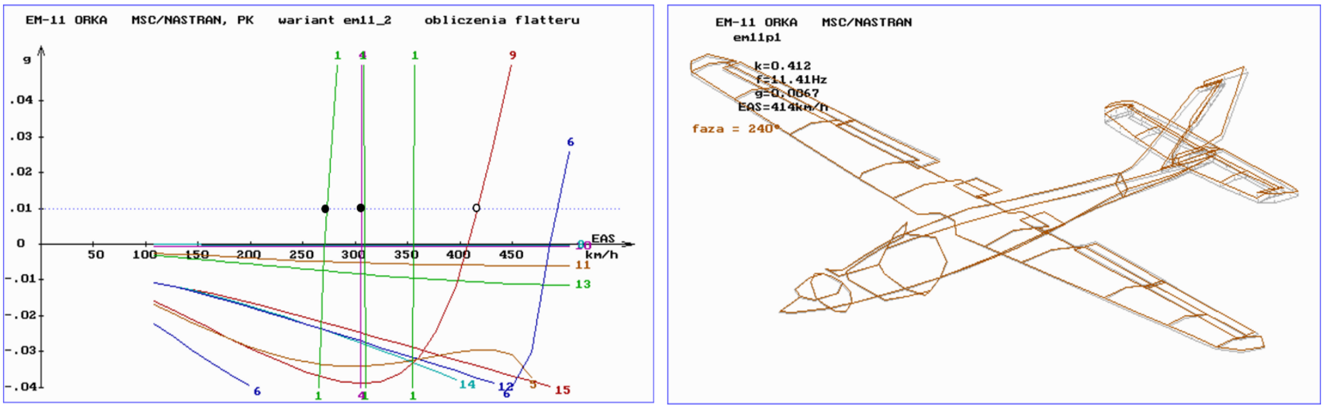


Figure 11 – Flutter damping curves and flutter mode for case 1.

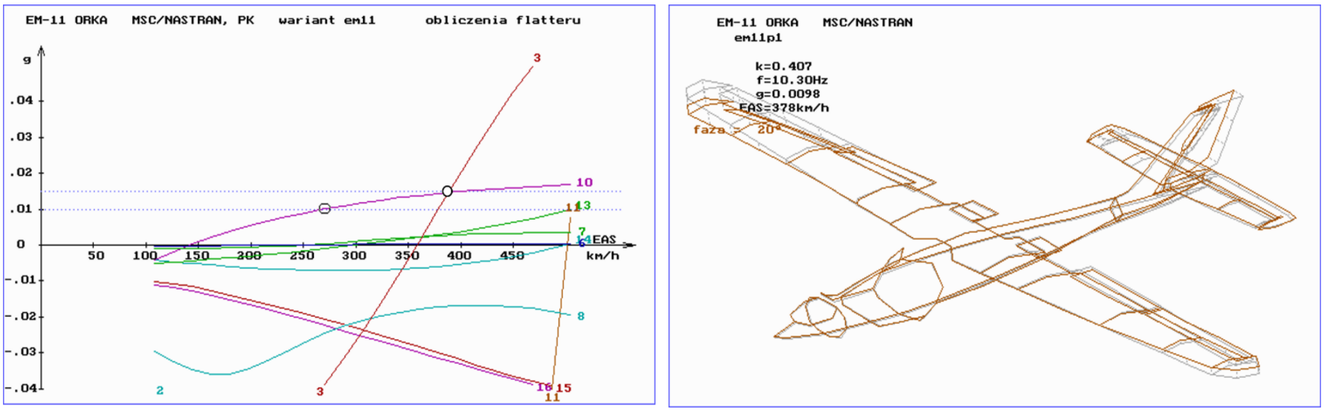


Figure 12 – Flutter damping curves and flutter mode for case 2.

4.2. Flutter results for MUT-OSA airplane

An example analysis of the flutter case for the MUT-OSA aircraft is presented in Fig. 13. The analysis was carried out using the natural vibration modes shown in Fig. 10. The graphs of the structural damping dependence on the speed g(V) refer to two calculation cases: 1) a model with unbalanced controls and 2) a model with balanced controls (the centres of mass of the rudder and elevator are then located on the hinge axis). In the case of the model with unbalanced controls, the tail flutter appears with dominant coupled modes 9 and 11 (shown in Fig. 10). The critical speed of self-excited vibrations for the assumed minimum level of structural damping g=0.01 is Vkr=108 km/h. Such a low flutter speed would be unacceptable for an aircraft designed to fly at speeds exceeding 200 km/h. Adding balancing masses, which cause the mass centres of the rudders to shift towards the hinge axis, proved to be sufficient to eliminate the risk of unwanted vibrations. In the second graph, all flutter curves run below the level of the assumed minimum damping (airplane is free from flutter danger).

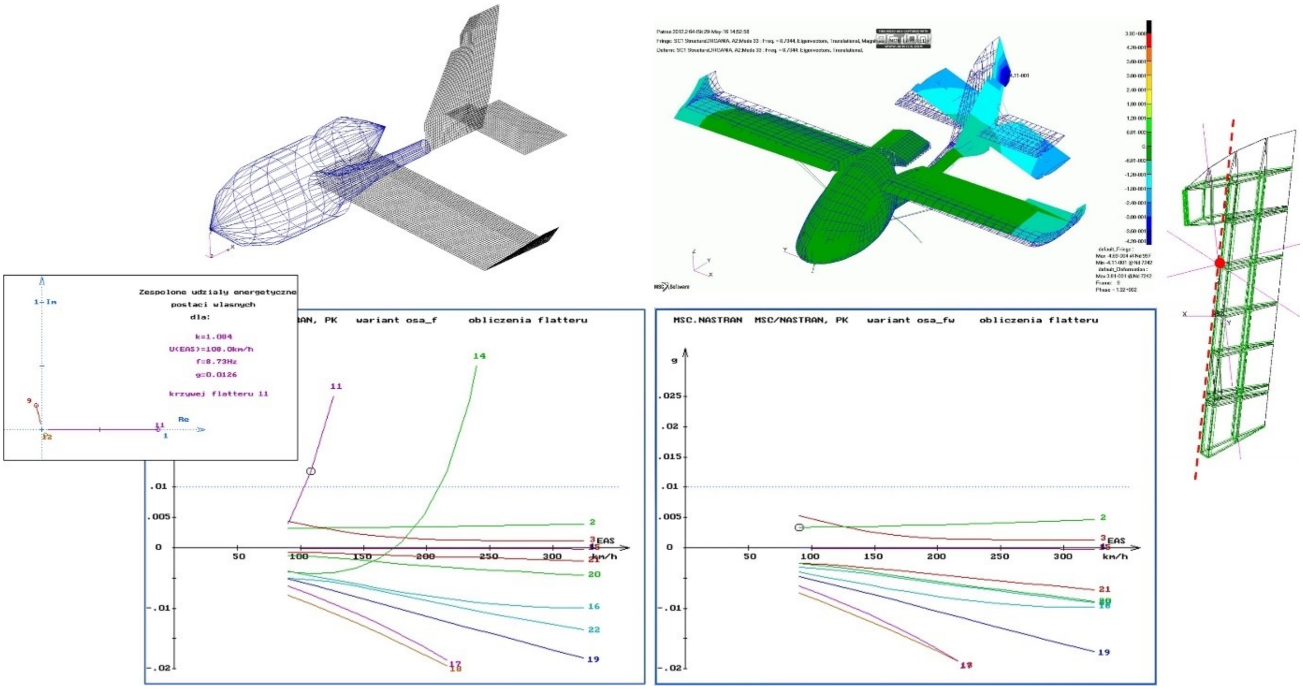


Figure 13 – Flutter analysis results for the OSA aircraft model: aerodynamic model, tail flutter vibration mode for the case with unbalanced control surfaces and g(V) curves graphs for the analyzed model cases: 1) unbalanced rudder and elevator, 2) mass-balanced both rudder and elevator.

5. Some methods of FE models validation

FEM models used for dynamic analyses in the field of aeroelasticity of structures should be characterized by appropriately selected mass properties and correct stiffness distributions. Correct selection of properties should be manifested by correct distribution of structural weight and appropriate compliance in response to established external excitation. However, structural models can be very complex in their construction. This complexity results from the need to reproduce often very complicated geometric shapes, to reflect the correct arrangement of structural elements, as well as to take into account additional elements approximating the masses of installations, equipment, consumables, loads and people. In order for the structural model to be fully useful for any more complex numerical analyses, it must be subject to verification and validation, as a result of which it will be possible to assess its usefulness. It is therefore necessary to define unambiguous criteria for the assessment of selected model properties in the light of their comparison with analogous features of the actual structure. The requirement to adapt model properties to the properties of the actual structure must be met so that the results of FEM analyses can be considered reliable and therefore acceptable. Significant deviations of numerical results from the results obtained during research and experimental tests will indicate that the model is not adapted and is useless.

There are many methods for checking and adapting model properties, most often based on comparison with the actual mechanical properties of the structure or with the results of measurements of selected physical quantities. In order to validate models at different stages of their creation, three comparative methods are proposed:

- comparison and adjustment of selected mass properties (component masses, mass moments of inertia of selected elements, location of the center of gravity);
- comparison and adjustment of stiffness properties by comparative analysis of displacements of the model and real structure;
- comparison and adjustment of dynamic properties by comparing the forms and frequencies of model natural vibrations with the resonant vibrations of the structure.

The simplest way to check selected structural properties of the model may be a comparative assessment of its component masses and the location of the center of gravity. In this method, the aim is to compare the values of the masses of the model's structural parts with the masses of real structural units. In the absence of real mass values, empirical expressions can be used, which are intended for estimating the component masses of the aircraft at the design level.

The second way to assess the quality of the model is to check its stiffness in a specific load direction. The operation consists in comparing the static displacements of the structure obtained for a specific load, and then adjusting the parameters affecting elasticity so that the model deforms identically to the real structure. The best way to carry out such a process is to conduct a static test and measure the displacements of the real structural parts mounted in the test stand. Analogous components of the model must be analyzed statically using the same load. After completing the experimental test and model analysis, the numerical displacements are compared with the displacements measured in the experiment. In the case of significant differences, it is necessary to correct the model properties by changing the values of the appropriate material constants (Young's modulus E, Kirchhoff's modulus G) or possibly by modifying the internal structural connections. The case of the wing of the EM-10 Orka business jet is given as an example of comparative analysis of stiffness. The real wing (actually a complete lifting airfoil) mounted on a laboratory stand was subjected to strength tests taking into account equivalent loads corresponding to the real loads of the aircraft in flight for point D of the load envelope. In turn, a static analysis was carried out for the aircraft wing model for support conditions and load values and distributions analogous to those from the test. The results in the form of wing displacement distributions along its stiffness axis obtained for three load levels (100%, 150% and 175%) are presented in the form of appropriate deflection lines on a comparative graph. The courses of experimental and model displacements are comparable, however, in this particular case it was not possible to achieve perfect agreement. The method of conducting the analysis and measurements and the deflection line graphs are shown in Fig. 14.

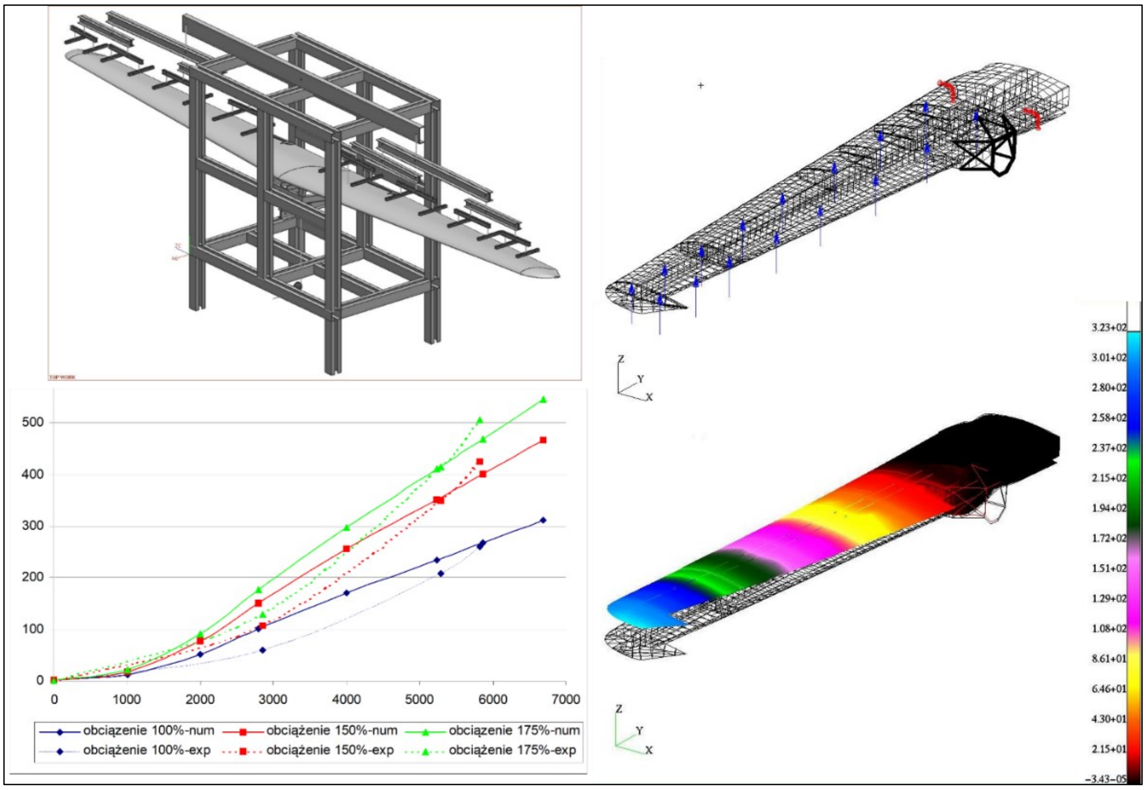


Figure 14 – Application of static strength test to validate the FE model of the EM-11 aircraft wing.

The most reliable method of assessing the dynamic properties of an aircraft structure is the ground resonance test (GVT). At the same time, the results of these tests are an excellent source of comparison for the results of the natural vibrations of the FEM model. However, the implementation of this type of test is quite troublesome due to technological and logistic reasons, therefore, such tests are often carried out in a limited scope or in a simplified manner. [8]

Resonance tests of aircraft described, among others, in items [11, 12] are a type of modal test during which the frequencies, forms and damping of natural vibrations induced on the structure in the resonance excitation mode are determined. For such measurements, the aircraft should be suspended on elastic cords, supported by pneumatic cushions or at least stand on tires with reduced pressure. A set of sensors (accelerometers) is placed on the aircraft at previously determined points. The aircraft is excited using electrodynamic exciters placed at design points where it is easy to force vibrations. The aircraft structure is excited to vibrate with an amplified sinusoidal signal generated by vibrators with a frequency gradually varying over the expected measurement range (usually between 2 and 50 Hz). Modal testing is terminated when the spectral transfer functions (FRFs) are identified. Typical output characteristics obtained after the measurement process are a stabilization diagram with possible amplification points corresponding to local maxima. The diagram provides a graphical representation of the system poles (eigenmodes) during different vibrational excitation modes of the structure. The summary spectral diagram obtained during MIMO excitation of the OSA aircraft is shown in Fig. 15.

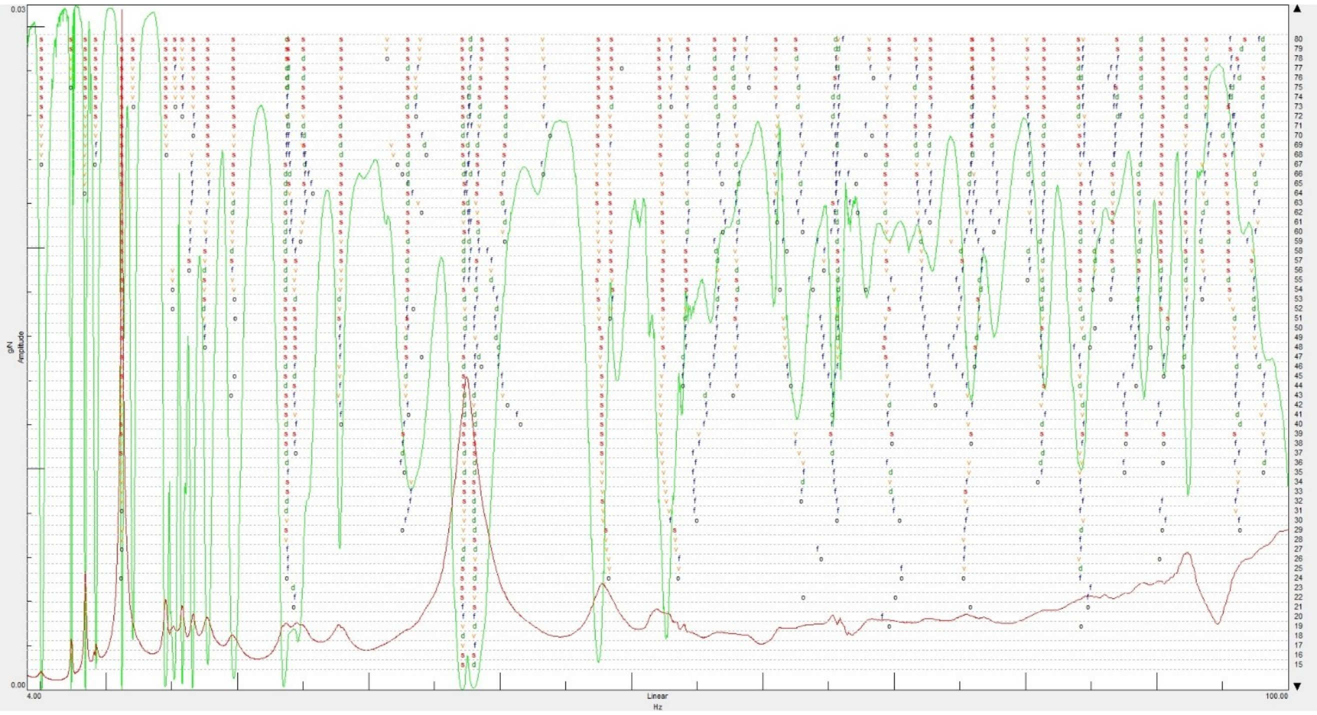


Figure 15 – Stabilization diagram according to the PolyMAX algorithm (generated in the LMS Test.Lab program) – the collective amplitude-frequency spectrum determined on the basis of the response of all measurement channels (red) and the course of the IMIF exciting force index (green).

6. Conclusions

Structural models intended for FEM analyses have specific features related to geometry, stiffness and mass. Their usefulness in numerical analyses is as valuable as they are properly adapted to real structures. In relation to the presented aircraft models developed and analysed in the FMAA MUT over the last dozen or so years, some key conclusions can be formulated, which would confirm the validity of conducting further analyses in the perspective of continuing research in the areas of dynamics and aeroelasticity of aircraft structures.

1. Structural models with mass and stiffness properties in combination with panel models for unsteady aerodynamics create an integrated model base for numerical flutter analyses in Nastran. Numerical analyses result in determining both critical flutter vibration parameters (critical velocity and coupled vibration frequency) and the possibility of visualizing vibration modes. The methodology, assuming significant model simplifications, gives satisfactory results in a relatively short time - it is possible to predict conditions favorable to flutter through the desired modification of the structural model. Computational results can be useful in the process of designing new structures, as well as in the process of overhauls and modernization of aging fleet aircraft.
2. The development of discrete models for multi-problem FEM analyses is a complicated and time-consuming process, requiring the implementation of many activities. The most important projects include, in the order of their execution:
   - modelling the geometry consistent with the actual aerodynamic body shape and dimensions,
   - modelling the internal structure while maintaining the reliability of the location, arrangement and dimensions of the elements,
   - establishing the material properties reliable for the structural materials,
   - taking into account the mass of internal non-structural elements through substitute elements representing the values of mass and mass moments of inertia,
   - using elements ensuring reliable structural connections,
   - ensuring reliable boundary conditions.
3. The validation of discrete models necessary for broader computational applications can be performed using the following techniques:

- correction of mass and stiffness properties based on comparison of model mass and displacement analyses results with experimental results,
- tuning of natural frequencies and modes to the results of GVTs.

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No. UGB 22-734/2024/WAT *Structural design of light airplanes in terms of optimizing strength and aeroelastic properties of the airframe*.

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