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FILTER-ERROR METHOD FOR ESTIMATION OF AERODYNAMIC PARAMETERS OF A SUBSCALE GENERIC FUTURE FIGHTER

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Abstract

The subscale flight testing method is a reduced-size aircraft, that reproduces free flight like a manned or full-scale aircraft. The Generic Future Fighter (GFF) is a subscale fighter airplane with a 14% scale. With the data collected in flight a system identification was done. Filter-Error Method was applied in this process. Results were also compared to Output-Error Method. Filter-Error Method considers both process noise and measurement noise. It is a stochastic approach and has proven to be adequate for parameter estimation in flight, under turbulent atmosphere. The main objective of this work is to demonstrate that FEM applied to subscale aircraft increases the accuracy of the estimated model, since these aircraft may be more susceptible to turbulence then full-scale aircrafts.

Keywords: Filter-Error method, Subscale, System Identification, Turbulence, Unmanned Aerial Vehicle

1. Introduction

The aerodynamic and structural optimization of fighter aircraft has pushed these airplanes to the limit. In order to achieve a greater advance in performance and control, innovative configurations must be developed [1]. One way to evaluate an unconventional aircraft, reducing risks and costs, is to test a model on a reduced scale [2]. The subscale flight testing method is a reduced-size aircraft and reproduces free flight (6 degree of freedom) like a manned or full-scale aircraft.
The most used technique for analysing flight dynamics is the construction of a simulation model [3]. To increase the accuracy of this model, data collected in flight is used in a system identification process to estimate the aerodynamic parameters.
An example of aerodynamic modelling, system identification and flight simulation of a subscale aircraft is the Airborne Subscale Transport Aircraft Research (AirSTAR). This aircraft model is part of a NASA effort to reduce the number of fatal accidents involving large transport aircraft. AirStar is a dynamically scaled aircraft that allows the use of flight test results from subscale aircraft to full-scale aircraft [8].
Flight test data of a subscale fighter aircraft will be presented in this work and a system identification process is applied to these data. The identification programs presented here were based on the algorithms proposed by Ravindra, a senior scientist at the German Aerospace Centre (DLR). Data analysis indicates that noise is present due to air turbulence and makes modelling the aircraft difficult [4]. Needing the application of methods to estimate the turbulence, which is a process noise.
The Output-Error Method (OEM) has historically proven adequate for identifying flight vehicles. However, in this work, the OEM, which is not able to estimate the noise caused by air turbulence, will be compared with the Filter-Error Method (FEM). The main objective of this work is to demonstrate that FEM applied to subscale aircraft increases the accuracy of the estimated model, since these aircraft may be more susceptible to turbulence then full-scale aircrafts.

2. Generic Future Fighter Subscale

In 2006 Saab Aeronautics, the Swedish Defence Research Agency (FOI), Volvo Aero, Linköping University and the Royal Institute of Technology (KTH) decided to develop a concept of a future fighter, the Generic Future Fighter GFF. The subscale model was built at Linköping University under the Future Aircraft Design and Demonstration (FADEMO) project. The subscale GFF has a wingspan of 1.47m, while the concept full-scale aircraft has a wingspan of 10.5 m.
The Generic Future Fighter (GFF) is a subscale platform, i.e. a 14% scaled model, Figure 1. A co-operation signed between the University of Linköping - Sweden - and Aeronautics Institute of Technology (ITA) -Brazil- allows the sharing of GFF flight test information. While the Swedish university is responsible for operating and developing test procedures and acquiring flight test data, ITA is responsible for implementing the system identification process with the acquired data. ITA already has experience with system identification in previous years [5].

Figure 1 – GFF subscale during flight tests at Sweden (Courtesy of Linköping University).

3. System Identification

Identification is an ensemble with experimental process and numerical analysis with the propose to obtain non-modeled, or uncertain parameters. The system identification process can be divided into 5 topics, maneuver, measurements, method, model and validation. Maneuver consist of applying a specific command to the aircraft's control inputs to excite a dynamic mode. The measurements are the variables acquired during the flight. The method is the mathematical or statistical tool to be used, such as the Output-Error Method and Filter-Error Method. The model are the mathematical equations that describe the aircraft's movement. Validation are the statistical tools that check the model's precision [7].
The FEM process, figure 2 a), is compared to the OEM in the figure 2 b). Where, z is the data measured during the flight and $\tilde{y}$ is the output vector of the system based on state estimator, the Extended Kalman filter (EKF). The OEM apply the integration of state equations to obtain the output vector, y, of the model.
Sobron (2021) states that the Filter-Error Method is the most appropriate algorithm for identifying aircraft models with the presence of atmospheric turbulence. The OEM is a simplified version of the Filter Error Method and is best suited for flight tests without atmospheric turbulence. The presence of turbulence is modeled as a process noise in the Filter-Error Method. For the OEM, turbulence can result in inaccurate estimates of model parameters and convergence problems [4].

3.1 Output-Error Method Theory

The OEM is an identification process that only considers measurement noise, which is a special case of the Filter-Error Method [4]. OEM seeks to adjust the parameters of a model in an iterative way,

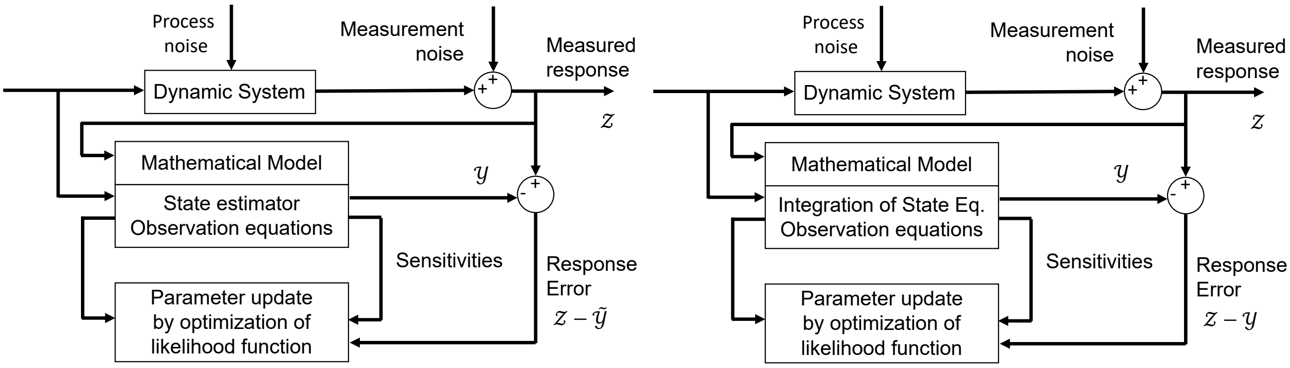

Figure 2 – a) Filter-Error Method. b) Output-Error Method Process [4].

making use of the minimization of the error between the measured variables and the output model. The optimal solution to this problem is made by minimizing the cost function:

$$J(\Theta) = det(R) \tag{1}$$

This equation above is a simplification of the maximum likelihood equation. Where R is the noise covariance matrix:

$$R = \frac{1}{N}\sum_{k=1}^{N}[z(t_k) - y(t_k)][z(t_k) - y(t_k)]^T \tag{2}$$

3.1.1 Model

The nonlinear mathematical model is given by:

$$\dot{x}(t) = f[x(t),\ u(t),\ \beta],\ x(t_0) = x_0 \tag{3}$$
$$y(t) = g[x(t), u(t), \beta] \tag{4}$$
$$z(t_k) = y(t_k) + G\,v(t_k) \tag{5}$$

Where f and g are nonlinear system functions, x is the state vector ($n_x \times 1$), u is the input vector ($n_u \times 1$) and y is the observation vector (or model output). β is the system parameters vector to be identified. The vector of measures z is sampled at N discrete points. The measurement errors (noise) are described by $v(t_k) = z(t_k) - y(t_k)$. The matrix G represent the measurement noise distribution matrices. Noise is supposed to be just summed and G is considered time-invariant.
The parameters to be estimated is shown below:

$$\Theta = [C_{D0}, C_{DV}, C_{D\alpha}, C_{L0}, C_{LV}, C_{L\alpha}, C_{m0}, C_{mV}, C_{m\alpha}, C_{mq}, C_{m\delta e}] \tag{6}$$

where $\Theta = [\beta]$. The coefficient C_{D0} is the drag coefficient for zero angle of attack, C_{DV} is the variation of drag coefficient with velocity, $C_{D\alpha}$ is the variation of drag coefficient with angle of attack, C_{L0} is the lift coefficient for zero angle of attack, C_{L0} is the variation of lift coefficient with angle of attack, C_{m0} is the pitching moment coefficient for zero angle of attack, C_{mV} is the variation of pitching moment coefficient with velocity, $C_{m\alpha}$ is the variation of pitching moment coefficient with angle of attack, C_{mq} is the variation of pitching moment coefficient with pitch rate and $C_{m\delta e}$ is the variation of pitching moment coefficient with elevator deflection angle.
In this model, the longitudinal aerodynamic parameters of the GFF subscale were estimated. The equations of state, of longitudinal motion, are described below:

$$\dot{V}_{TAS} = -\frac{\bar{q}S}{m}C_D + g\,sin(\alpha - \theta) + \frac{F_e}{m}cos(\alpha + \sigma_T) \tag{7}$$

$$\dot{\alpha} = -\frac{\bar{q}S}{mV}C_L + q + \frac{q}{V}cos(\alpha - \theta) - \frac{F_e}{mV}sin(\alpha + \sigma_T) \tag{8}$$

$$\dot{\theta} = q \tag{9}$$

$$\dot{q} = \frac{\bar{q}S\bar{c}}{I_{YY}}C_m + \frac{F_e}{I_{YY}}(l_{tx}sin(\sigma_T) + l_{tz}cos(\sigma_T)) \tag{10}$$

where $\dot{V}_{TAS}$ is the true air speed, $\bar{q}$ is the dynamic pressure, S is the reference area, m is the mass, C_D is the drag coefficient, g is the gravity, α is the angle of attack, θ is the pitch angle, F_e is the thrust force, σ_T is the angle between thrust direction and longitudinal reference axis of the airplane, V is the speed, C_L is the lift coefficient, q is the pitch rate, $\bar{c}$ is the mean aerodynamic chord, C_m is the pitching moment, I_{YY} is the moment of inertia and l_{tx} l_{tz} is the location of engines from the center of gravity. The lift, drag and pitching moment coefficients are described as follows:

$$C_D = C_{D0} + C_{DV}\frac{V}{V_0} + C_{D\alpha}\alpha \tag{11}$$

$$C_L = C_{L0} + C_{LV}\frac{V}{V_0} + C_{L\alpha}\alpha \tag{12}$$

$$C_m = C_{m0} + C_{mV}\frac{V}{V_0} + C_{m\alpha}\alpha + C_{mq}\frac{q\bar{c}}{2V_0} + C_{m\delta e}\delta e \tag{13}$$

where V_0 is the reference speed, for a steady-state condition. The equations 11, 12 and 13 present the aerodynamic parameters to be identified, according to the vector of parameters 6.

3.1.2 Optimization Method

To determine the minimum of the cost function, the Taylor series expansion is applied to the cost function until the second term and then equal to zero:

$$\left(\frac{\partial J}{\partial \Theta}\right)_{i+1} \approx \left(\frac{\partial J}{\partial \Theta}\right)_i + \left(\frac{\partial^2 J}{\partial \Theta^2}\right)_i \Delta\Theta \tag{14}$$

$$\left(\frac{\partial J}{\partial \Theta}\right)_i + \left(\frac{\partial^2 J}{\partial \Theta^2}\right)_i \Delta\Theta = 0 \tag{15}$$

Isolating $\Delta\Theta$:

$$\Delta\Theta = -\left[\left(\frac{\partial^2 J}{\partial \Theta^2}\right)_i\right]^{-1}\left(\frac{\partial J}{\partial \Theta}\right)_i \tag{16}$$

At each iteration the parameter values are updated as follows:

$$\Theta_{i+1} = \Theta_i + \Delta\Theta \tag{17}$$

The partial derivative of the cost function is defined as:

$$\frac{\partial J}{\partial \Theta} = -\sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial \Theta}^T\right] R^{-1}\left[z(t_k) - y(t_k)\right] \tag{18}$$

The second partial derivatives, considering the Gauss-Newton method, it is equal to:

$$\frac{\partial^2 J}{\partial \Theta^2} \approx -\sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial \Theta}^T\right] R^{-1}\frac{\partial y(t_k)}{\partial \Theta} \tag{19}$$

3.2 Filter-Error Method Theory

The application of the Output-Error Method is limited to flight tests without atmospheric turbulence. The presence of turbulence, process noise, makes it difficult to estimate parameters and the Output-Error Method presents imprecise estimates and convergence problems.

3.2.1 Model

The state space mathematical model is given by the stochastic equations:

$$\dot{x}(t) = f[x(t),\ u(t),\ \beta] + Fw(t),\ x(t_0) = x_0 \tag{20}$$
$$y(t) = g[x(t), u(t), \beta] \tag{21}$$
$$z(t_k) = y(t_k) + G\,v(t_k) \tag{22}$$

where f and g are vector functions of dimensions n_x and n_y, x is the state vector $(n_x \times 1)$, u is the input vector $(n_u \times 1)$ and y is the observation vector (or model output). β is the system parameters vector to be identified . The vector of measures z is sampled at N discrete points. Process noise $w(t)$ and measurement noise $v(t_k)$ are uncorrelated and are mutually independent. The matrices F and G represent the measurement and process noise distribution matrices. Noise is supposed to be just summed and the noise distribution matrices F and G are considered time-invariant and independent of each other.

3.2.2 FEM algorithm

The estimates of the parameter vector Θ and the matrix R are obtained by minimizing the likelihood function:

$$J(\Theta) = det(R) \tag{23}$$

Where $\tilde{y}$ is the output of the model based on the predicted states. The variable z is the vector of the measured data with N discretized time samples. Where R is the steady-state covariance matrix described by:

$$R = \frac{1}{N}\sum_{k=1}^{N}[z(t_k) - \tilde{y}(t_k)][z(t_k) - \tilde{y}(t_k)]^T \tag{24}$$

A summary of the Filter-Error-Method calculation steps for non-linear systems is presented below.
Initially the equations of states are integrated using the 4th order Runge-Kutta method. Then, the observation variables are calculated according to the equation:

$$\tilde{y} = g\,[\tilde{x}(t_k), u(t_k), \beta] + b_y \tag{25}$$

Where b_y is the output bias. Then the residuals $[z(t_k) - \tilde{y}(t_k)]$ and the maximum likelihood estimate of R are computed.
The next step is to solve the steady-state Riccati equation for P. The first-order approximation of the steady-state Riccati equation is equal to:

$$AP + PA^T - \frac{1}{\Delta t}PC^TR^{-1}CP + FF^T = 0 \tag{26}$$

Where A and C are the linearized state space matrices. F is the process noise distribution matrix and P is the covariance matrix of the state-prediction error.
With the value o P is possible to compute the gain matrix:

$$K = PC^TR{-}1 \tag{27}$$

Thus, with the calculated Kalman gain matrix K, the states are corrected as follows:

$$\hat{x}(t_k) = \tilde{x}(t_k) + K[z(t_k) - \tilde{y}(t_k)] \tag{28}$$

The response gradients from perturbed system equations are numerically approximated:

$$\left(\frac{\partial \tilde{y}}{\partial \Theta}\right)_{ij} \approx \frac{\tilde{y}_i(\Theta + \delta\Theta_j e^j) - \tilde{y}_i(\Theta)}{\delta\Theta_j} \tag{29}$$

Then the calculation of updated parameters $\Delta\Theta$ applying Gauss-Newton method is performed. Where the update of parameters is defined as:

$$\Theta_{i+1} = \Theta_i + \Delta\Theta \tag{30}$$

$$F\ \Delta\Theta = -G \tag{31}$$

Finally, it is iterated until convergence. The optimization method to determine the minimum cost function is the same method used in OEM.

4. Results

The Figure 3 presents time histories of seven output variables, true airspeed, angle of attack, pitch angle, pitch rate, the derivative of the pitch rate, acceleration in the x-axis and acceleration in the z-axis. The last graph shows the elevator input. The true airspeed and angle of attack are two variables related to air flow, that are likely to suffer more interference from turbulence. In this figure, the Output-Error Method was used for model estimation, where the red line represents the estimated model and the blue line represents the data measured in flight.

The same flight phase of the GFF is also presented in the Figure 4. However, the red curve presents the model estimated with the Filter-Error Method, where turbulence is considered in the prediction of the parameters. Comparing the two figures, it can be seen that the result of the Filter-Error Method was closer to the data collected in flight. To quantify this comparison we can analyze the cost function of each method. The cost function, $det(R)$, of the OEM was equal to 4.4602e-07, after convergence of the algorithm. And the cost function, $det(R)$, of the FEM was equal to 8.5254e-18, after convergence. A table with the results is presented below:

Table 1 – FEM and OEM Results

	C_{D0}	C_{DV}	$C_{D\alpha}$	C_{L0}	C_{LV}	$C_{L\alpha}$	C_{m0}	C_{mV}	$C_{m\alpha}$	C_{mq}	$C_{m\delta e}$
FEM	0.0250	-0.0196	0.5516	-0.2360	0.1888	3.5164	0.0343	-0.0172	-0.1977	0.9559	-0.1592
OEM	0.0140	-0.0004	0.3749	-0.2208	0.1978	3.0000	0.0370	-0.0164	-0.1978	-1.5774	-0.3024

5. Conclusion

The subscale method for fighter design can add information and model the aircraft in some regions of the flight envelope. It is an important tool for analysing and developing aircraft with non-conventional configurations. Considering the results presented, to ensure a good accuracy of the model, the use of the Filter-Error Method proved to be more suitable for subscale fighter applications. These results are part of an ongoing research, where other maneuvers will be identified and used to validate the model.

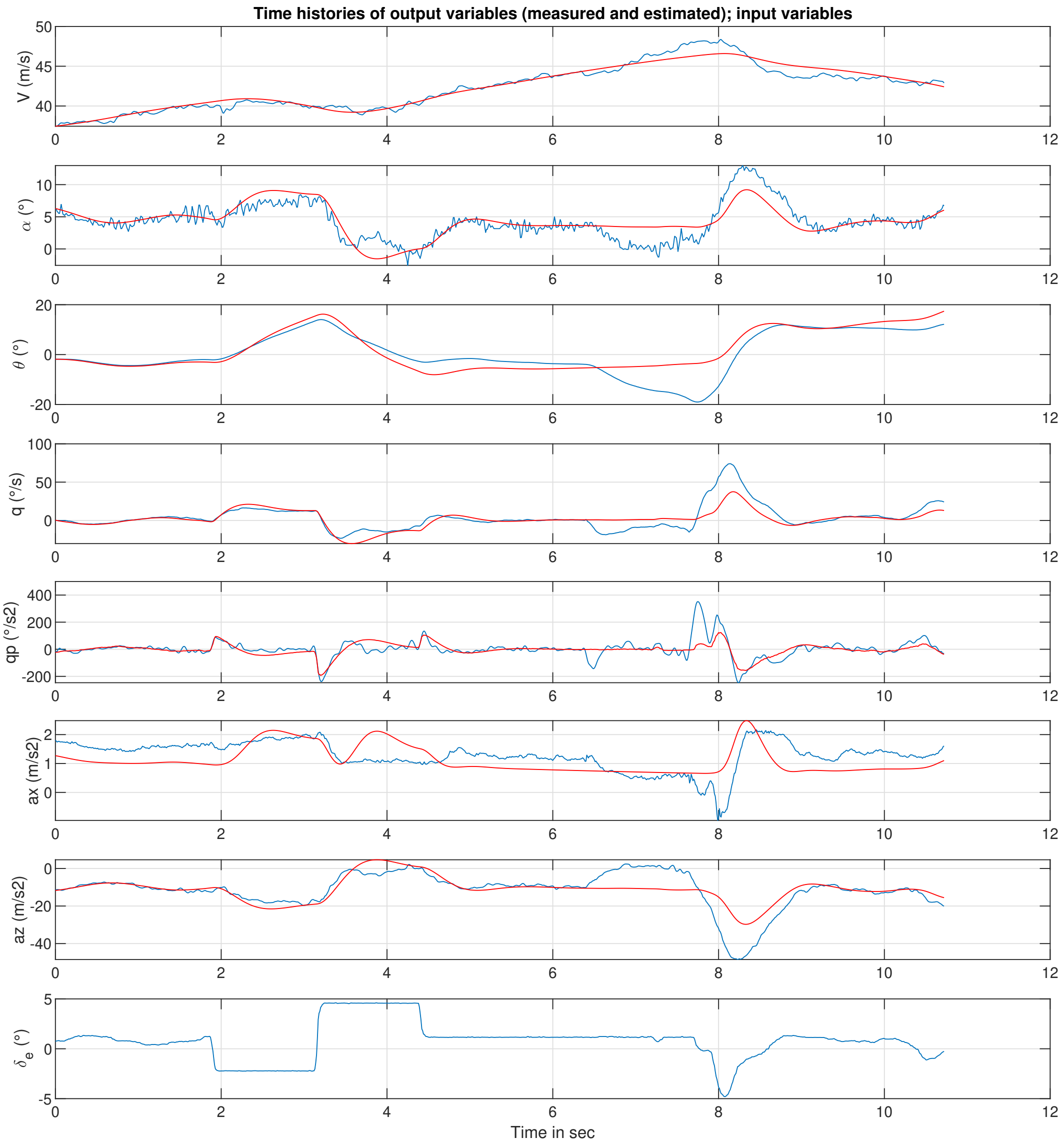

Figure 3 – Output-Error Method Results

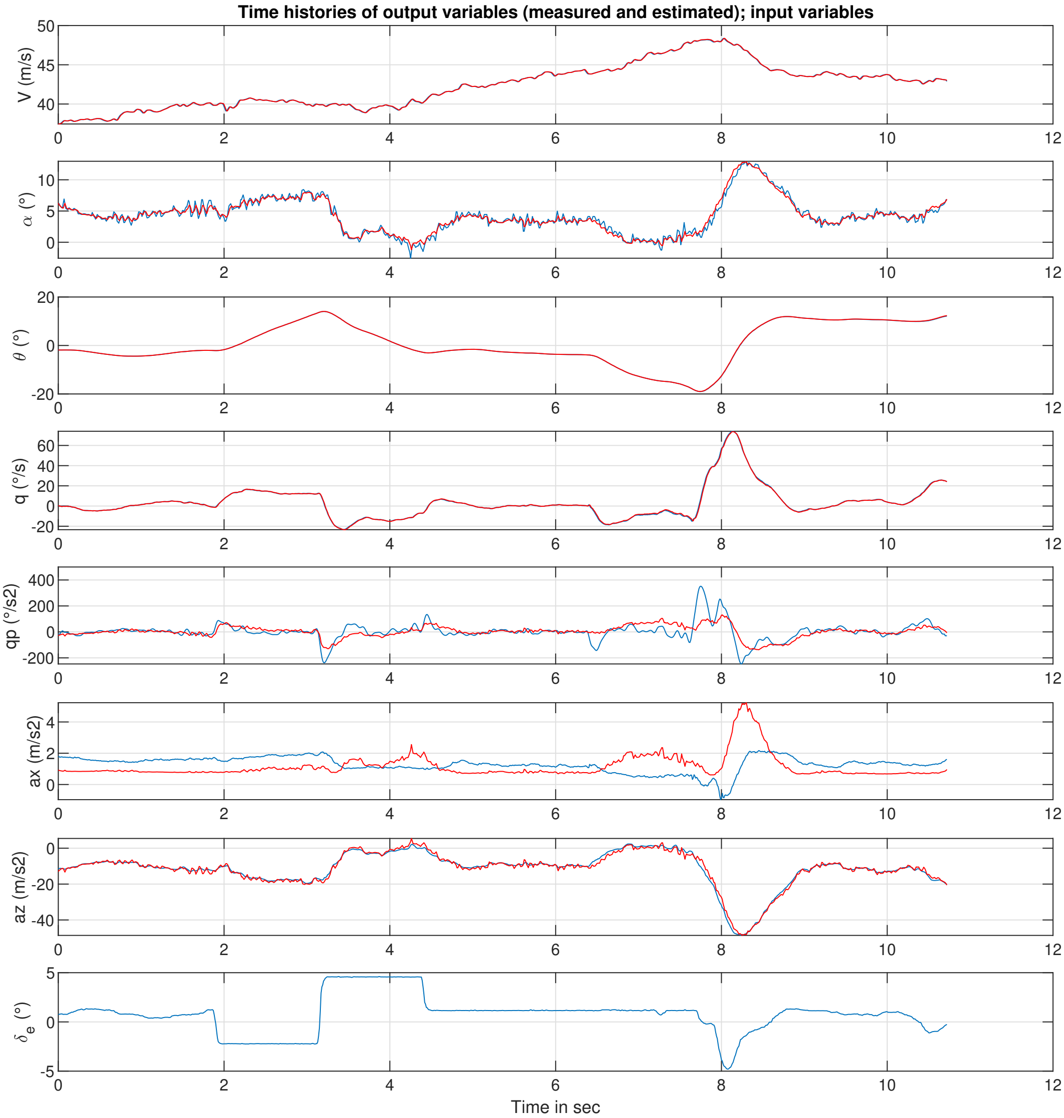

Figure 4 – Filter-Error Method Results

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7. Copyright Statement

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