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SPACE DEBRIS CLOUD PROPAGATION THROUGH PHASE SPACE DOMAIN BINNING

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Abstract

It is estimated that almost one million debris greater than 1 cm currently orbit the Earth, posing hazard to operational satellites. Therefore, the traditional piece-by-piece approach, to monitor the evolution of such small space debris, is computationally prohibitive. This problem is here addressed through an analogy with fluid dynamics, considering the population of fragments as a cloud, whose density is numerically propagated in the phase space of Keplerian elements. The main goal of this study is to develop a model that is able to deal with fragmentation in any orbital region and under any dynamical regime. This objective is achieved by combining the method of characteristics, applied to the continuity equation, with the phase space splitting into bins.

Keywords: Space debris, Continuum approach, Density propagation, Binning

1. Introduction

The growing dependence of our daily lives on space services has caused a massive growth in space activities over the past decade. While it sounds fascinating, the ever-growing population of objects orbiting the Earth could be detrimental to future space missions. It is estimated that more than one million debris objects greater than 1 cm currently orbit the Earth [1], posing a hazard to operational satellites. The traditional piece-by-piece approach, to monitor the evolution of such small space debris, is not feasible from a computational point of view. This problem has been elegantly addressed through an analogy with fluid dynamics. At first, the small fragments were modelled in terms of their spatial density [2], whose evolution in time was obtained thanks to the continuity equation. In [3], [4], the Method Of Characteristics (MOC) was adopted to find analytical solutions to the continuity equation in the phase space of a subset of Keplerian elements, under simplified orbital regimes, which limited the analysis to fragmentations in Low-Earth-Orbits (LEO). The Starling suite was developed at Politecnico di Milano [5] in the framework of the continuum approach, to deal with debris fragments propagation of any dimension and subjected to non-linear dynamics. This approach allowed to potentially extend the continuum approach to any orbital regime. In [5], a Gaussian Mixture Model (GMM) was selected as density interpolation technique; reference hypersurfaces were used to map the density distribution into a modified phase space, more suitable to be represented by Gaussian distributions. As specified in [5], the method is not currently able to accommodate forces that lead to resonances on a small subset of the phase space, as it could be the case of third-body perturbation or solar radiation pressure. Such resonances tend to separate, or branch out, parts of the distribution from the main bulk of the characteristics.

This work aims at developing a method, for propagation and interpolation of the fragments' cloud density, able to deal with any orbital regime. A binning approach is adopted for the density interpolation in the six-dimensional phase space of Keplerian elements and area-to-mass ratio. The MOC is adopted for the propagation of the debris density under orbital perturbations. The reference characteristics to be propagated are sampled from a domain defined according to probabilistic considerations on the way fragments distribute in the phase space.

2. Estimation of the initial density distribution

This section describes the method adopted for computing the initial density distribution. The idea is to define, through probabilistic considerations, the phase space domain that the fragments will occupy after the fragmentation event and to estimate the density in all regions belonging to it, through a binning approach. In other words, the density is evaluated in each bin belonging to the domain; if the grid is fine enough, despite of the discontinuous nature of the method, the computed density distribution well represents the initial cloud of fragments. The characteristics to be propagated through the MOC are eventually sampled from the estimated fragments' density. This approach allows both to strongly reduce the computational time and to improve the accuracy of the model, since the density is evaluated only in those regions that are most likely to have fragments.

2.1 Probabilistic definition of the initial domain

The definition of the initial domain proposed in this paper depends on the model adopted for estimating the fragments' spreading due to a fragmentation event. Indeed, it relies on a probabilistic analysis on the likelihood for a debris to reach a region of the phase space. The breakup model adopted for this study is the reformulated NASA Standard Breakup Model (SBM) proposed in [6]. The NASA SBM is reformulated as a probability distribution function dependent on the fragments characteristic length L, area-to-mass ratio A/m and the ejection velocity Δv, which is assumed isotropically distributed in direction. The model, which relies on both historical orbital data and ground-based impact tests, assumes the following probability functions:

$$p_\lambda = \log(10)\beta \frac{10^{-\beta\lambda}}{10^{-\beta\lambda_0 - 10^{-\beta\lambda_1}}} \tag{1a}$$

$$p_{\chi|\lambda} = \sum_i \alpha_i(\lambda) \mathcal{N}\left(\mu_\chi^{(i)}(\lambda), \sigma_\chi^{(i)}(\lambda)\right) \qquad \sum_i \alpha_i(\lambda) = 1 \tag{1b}$$

$$p_{\nu|\chi} = \mathcal{N}(\mu_\nu(\chi), \sigma_\nu(\chi)) \tag{1c}$$

where β is a unitless parameter dependent on the type of fragmentation; λ, χ and ν are the logarithms to base 10 of the characteristic length L, area-to-mass ratio A/M and ejection velocity Δv, respectively; λ_0 and λ_1 are the logarithms to base 10 of lower L_0 and upper L_1 boundaries on the characteristic length; $\mu_\chi^{(i)}$, $\sigma_\chi^{(i)}$, μ_ν and σ_ν are mean and standard deviation of normal distributions $\mathcal{N}$ in χ and ν, which depends on the type of fragmentation; α_i are factors to weight the relative importance of the normal distributions in the conditional probability in χ dependent on λ.
The threshold values for the logarithm to base 10 of area-to-mass ratio, χ, and ejection velocity, ν, are selected according to the related cumulative density functions CDF_χ and $CDF_{\nu|\chi}$. Indeed, since the area-to-mass ratio is included in the set of variables defining the phase space, the proposed model firstly defines the grid in χ according to the cumulative density function CDF_χ, and secondly computes the domains in Keplerian elements, for each bin in area-to-mass ratio, according to $CDF_{\nu|\chi}$. In other words, the domain in Keplerian elements probabilistically reachable by fragments varies depending on the range in area-to-mass ratio considered.
The cumulative density function in χ can be computed through marginalisation over λ, as follows:

$$CDF_\chi = \int_{-\infty}^{\chi} \int_{\lambda_0}^{\lambda_1} p_{\chi|\lambda} p_\lambda \mathrm{d}\lambda \mathrm{d}\chi \tag{2}$$

The threshold value in χ is computed as:

$$\chi_\xi = CDF_\chi^{-1}(\xi) \tag{3}$$

where ξ is a factor ranging from 0 to 1, which can be tuned according to targeted level of accuracy. As already mentioned, the range in ν is defined according to the conditional cumulative density function $CDF_{\nu|\chi}$, which is simply the CDF of a normal distribution. Therefore, the following method is adopted:

$$\nu_{\eta} = CDF^{-1}_{\nu|\chi}(\eta) = \mu_{\nu}(\chi) + k(\eta)\sigma_{\nu}(\chi) \tag{4}$$

where η is a factor ranging from 0 to 1 and $k(\eta)$ is a dependent variable, which tunes the displacement in ν with respect to the mean value μ_{ν} according to η.
The domain in area-to-mass ratio and ejection velocity must be converted into a domain in area-to-mass ratio and Keplerian elements. The objective is to compute the maximum variation in Keplerian elements, with respect to the fragmentation point, associated to the ejection velocities that satisfy the condition:

$$\Delta v \leq 10^{\nu_{\eta}} \tag{5}$$

where ν_{η} is the threshold value defined in Eq. (4). Therefore, in the following, the variation of the Keplerian elements due to an ejection velocity Δv will be derived, as function of two angles γ and ϕ, which define the direction of the impulse. The ejection velocity vector is here defined in the radial–transversal–out-of-plane (RSW) reference frame, as follows.

$$\boldsymbol{\Delta v} = \{\Delta v \cos\gamma \cos\phi, \Delta v \sin\gamma \cos\phi, \Delta v \sin\phi\}^{T} \tag{6}$$

where γ and ϕ are the in-plane and out-of-plane angles, respectively. When the fragmentation occurs, the generated fragments are assumed to share the same initial position, but to have a velocity that depends on the impulse they received. Therefore, the fragments are distributed according to a 4D density function in velocity and area-to-mass only. When moving to a distribution in Keplerian elements, the same dimensionality must be preserved; hence, the trasformed distribution will be in a subset of three Keplerian elements and area-to-mass ratio [7]. The remaining dependent Keplerian elements are function of the independent ones and of the fragmentation point location. The same concept can be analysed from anther perspective: since no uncertainty on the initial position is considered, the orbits of the generated fragments must intersect the orbit of the parent object in the fragmentation point. This means that the new orbits have only three degree of freedom, that can freely vary in the domain that satisfies the constraint on velocity expressed in Eq. (5). Hence, only the variations of semi-major axis a, eccentricity e and inclination i, due to the ejection velocity vector of Eq. (6), will be computed for defining the initial domain.
The variation of the semi-major axis Δa is computed from the energy equation, as follows.

$$\Delta a(\Delta v, \gamma, \phi) = \frac{\mu r}{2\mu - r\left(v + \Delta v_m(\Delta v, \gamma, \phi)\right)^2} - a \tag{7}$$

where μ is the planetary constant of the Earth; r and v are the position and velocity modules of the parent object at the fragmentation epoch; a is the semi-major axis of the parent object orbit; Δv_m is the variation of the velocity module due to the fragmentation event, defined as follows.

$$\Delta v_m(\Delta v, \gamma, \phi) = \sqrt{(v_t + \Delta v \cos\gamma \cos\phi)^2 + (v_r + \Delta v \sin\gamma \cos\phi)^2 + (\Delta v \sin\phi)^2} - v \tag{8}$$

where v_t and v_r are the transversal and radial components of the velocity of the parent object at the fragmentation epoch.
The eccentricity vector of a fragment after the fragmentation can be computed as:

$$\boldsymbol{e} + \boldsymbol{\Delta e} = \frac{1}{\mu}(\boldsymbol{v} + \boldsymbol{\Delta v}) \times (\boldsymbol{h} + \boldsymbol{\Delta h}) - \frac{\boldsymbol{r}}{r} \tag{9}$$

where $\boldsymbol{h}$ and $\boldsymbol{\Delta h}$ are the angular momentum of the parent object orbit and its variation due to the fragmentation, respectively, whose sum is defined as follows.

$$(\boldsymbol{h} + \boldsymbol{\Delta h}) = \boldsymbol{r} \times (\boldsymbol{v} + \boldsymbol{\Delta v}) \tag{10}$$

where, again, the position vector $\boldsymbol{r}$ is kept unchanged, since no uncertainty on the initial position is assumed. The difference between the norm of the eccentricity vector of Eq. (9) and the eccentricity of the parent object orbit e allows the computation of the eccentricity variation Δe due to a variation of velocity according to the ejection velocity vector of Eq. (6).

$$\Delta e(\Delta v, \gamma, \phi) = ||\boldsymbol{e} + \boldsymbol{\Delta e}|| - e \tag{11}$$

The variation of the inclination Δi is computed as follows.

$$\Delta i(\Delta v, \gamma, \phi) = \arccos\left(\frac{h_z^{in.} + \Delta h_z^{in.}}{h + \Delta h}\right) - i \tag{12}$$

where $h_z^{in.}$ and $\Delta h_z^{in.}$ are the out-of-plane components of the angular momentum of the parent object orbit and its variation due to the fragmentation in the inertial reference frame, whose sum is computed as follows.

$$h_z^{in.} + \Delta h_z^{in.} = R_3(\Omega) R_1(i) R_3(\omega + f)(\boldsymbol{h} + \boldsymbol{\Delta h}) \cdot \widehat{\boldsymbol{k}} \tag{13}$$

where i, Ω, ω and f are inclination, right ascension of ascending node, argument of periapsis and true anomaly of the parent object orbit at fragmentation epoch; $\widehat{\boldsymbol{k}}$ is the unitary vector in the direction of the Z axis of the inertial frame; R_1 and R_3 are the rotation matrices about x and z axes; $\boldsymbol{h} + \boldsymbol{\Delta h}$ is the angular momentum of the orbit of a generic fragment in the RSW reference frame.

So far, the variations of semi-major axis, eccentricity and inclination depend on the in-plane and out-of-plane angles of the ejection velocity vector. In order to define the sub-domains in Keplerian elements associated to each A/M bin, the dependency on the two angles must be filtered out. One approach could be to compute the maximum variation of the elements, given Δv; however, this would cause a significant growth of the domain, thus covering regions of the phase space with an extremely low density value. Indeed, the probability density function in such a region is further scaled by a factor $1/2\pi^2$, because that variation in keplerian element can be achieved with a single combination of γ and ϕ. If factors ξ and η of Eqs. (3) and (4) are chosen close to 1, the sub-domains can be more conveniently computed according to the maximum average variation of the Keplerian elements, as follows.

$$\overline{\Delta\alpha_i^+} = \max_{\Delta v} \overline{\Delta\alpha_i^+}(\Delta v) = \max_{\Delta v} \frac{1}{(\gamma_{2_i}^+ - \gamma_{1_i}^+)(\phi_{2_i}^+ - \phi_{1_i}^+)} \int_{\gamma_{1_i}^+}^{\gamma_{2_i}^+} \int_{\phi_{1_i}^+}^{\phi_{2_i}^+} \Delta\alpha_i(\Delta v, \gamma, \phi) \mathrm{d}\phi \mathrm{d}\gamma \tag{14a}$$

$$\overline{\Delta\alpha_i^-} = \max_{\Delta v} \overline{\Delta\alpha_i^-}(\Delta v) = \max_{\Delta v} \frac{1}{(\gamma_{2_i}^- - \gamma_{1_i}^-)(\phi_{2_i}^- - \phi_{1_i}^-)} \int_{\gamma_{1_i}^-}^{\gamma_{2_i}^-} \int_{\phi_{1_i}^-}^{\phi_{2_i}^-} \Delta\alpha_i(\Delta v, \gamma, \phi) \mathrm{d}\phi \mathrm{d}\gamma \tag{14b}$$

where $\Delta\alpha_i$ is the variation of the i^{th} Keplerian element and the plus and minus signs indicate the regions in the γ–ϕ domain that lead to a positive and negative variation of the Keplerian element α_i, respectively.

When the ejection velocity Δv is higher than the impulse needed to escape the Earth gravity field, some combinations of γ and ϕ lead to a singularity in the variation of the semi-major axis Δa and negative (i.e., hyperbolic) semi-major axis values for the fragments' orbit. This causes the average semi-major axis variation $\overline{\Delta a}$ to acquire a meaningless value; therefore, Δa is computed from the definition of the module of the angular momentum in terms of Keplerian elements, as follows.

$$h + \Delta h = \sqrt{\mu(a + \widetilde{\Delta a})\left(1 - (e + \overline{\Delta e^{\pm}})^2\right)} \tag{15}$$

where $\overline{\Delta e^{\pm}}$ is the average variation of eccentricity, which might be either positive or negative, depending on the angles γ and ϕ. Eq. (15) can be solved for $\widetilde{\Delta a}$, which leads to an expression of the kind:

$$\widetilde{\Delta a} = \widetilde{\Delta a}(\Delta v, \gamma, \phi, \overline{\Delta e^{\pm}}) \tag{16}$$

The approximated variation of the semi-major axis $\widetilde{\Delta a}$ has a singularity when the fragments, subjected to an ejection velocity Δv are, on average, injected on hyperbolic trajectories. The average variation of the semi-major axis, in positive and negative directions, can be computed according to Eqs. (14), by adopting the local approximated semi-major variation of Eq. (16).

2.2 Monte Carlo integration and density averaging

Once the initial domain is defined, the density distribution is estimated through a binning approach. The phase space domain is partitioned into bins and the density is averaged over the bin volume. To this aim, the probability density function in ejection velocity and area-to-mass is transformed into a density function in the subset of Keplerian elements (a, e and i), through change of variables [7], as follows.

$$p_{\boldsymbol{\alpha},A/M} = \frac{p_{\boldsymbol{\Delta v},A/M}\left(\varphi_{\boldsymbol{\alpha s}}^{-1}(\boldsymbol{\alpha})\right)}{\det J_{\boldsymbol{\alpha s}}^{3\times 3}} \tag{17}$$

where $\varphi_{\boldsymbol{\alpha s}}$ is the transformation from Keplerian $(\boldsymbol{\alpha})$ to Cartesian $(\boldsymbol{s})$ coordinates, with the Jacobian $J_{\boldsymbol{\alpha s}}$ defined as:

$$J_{ij} = \frac{\partial \varphi_i}{\partial x_j} \tag{18}$$

The average fragments density in each bin is computed as:

$$\overline{n_i} = \frac{N}{V_{\boldsymbol{\alpha}}^i\ \delta A/M^i} \int_{A/M_1^i}^{A/M_2^i} \int_{V_{\boldsymbol{\alpha}}^i} p_{\boldsymbol{\alpha},A/M}\, \mathrm{d}\boldsymbol{\alpha}\mathrm{d}A/M \tag{19}$$

where $\overline{n_i}$ is the density in the i^{th} bin; $V_{\boldsymbol{\alpha}}^i$ is the volume in Keplerian elements of the bin; $\delta A/M^i$ is the range in area-to-mass ratio of the bin; N is the total number of fragments generated by the fragmentation event [8]. In this study, an equally sized binning is adopted; hence, the volume of each bin is constant throughout the whole phase space domain.

Adopting a binning approach, the density varies discretely through the domain; however, if the grid is fine enough, the bins density is able to well describe the shape of the density distribution. Therefore, it is of paramount importance to properly define the step-size in each dimension of the phase space. Since the density is averaged at bin level, the smaller is the variation of the density over the bin volume, the more accurate the approximated fragments' density is. Each element of the gradient of the density with respect to the Keplerian elements indicates how fast the density is changing locally; thus, it can be used as an indicator of how small the step-size in each Keplerian element $\delta\alpha_i$ should be to grasp the variation of the density in the phase space. Since an equally-sized binning approach is adopted, the average module of the gradient of the density is chosen as reference for defining the step-size in semi-major axis, eccentricity and inclination. It is computed as follows.

$$\overline{\left|\frac{\partial n}{\partial \Delta\alpha_i}\right|} = \frac{N}{2\pi^2 \Delta v} \int_0^{\Delta v} \left|\frac{\partial p_{\nu|\chi}}{\partial \Delta v}\right| \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \left|\frac{\partial \Delta\alpha_i}{\partial \Delta v}\right|^{-1} \mathrm{d}\phi \mathrm{d}\gamma \mathrm{d}\Delta v \tag{20}$$

where the derivatives in Eq. (20) are computed according to Eqs. (1c), (11), (12) and (15). The following heuristic relation between the step-sizes and the derivatives of the density with respect to the Keplerian elements is adopted:

$$\overline{\left|\frac{\partial n}{\partial \Delta\alpha_i}\right|}\ \delta\alpha_i \approx \frac{n_{\max}}{r} \tag{21}$$

where r is a factor that can be tuned according to the desired level of accuracy. Note that the left-hand side of Eq. (21) is an estimate of the average change of density value associated to a step in the Keplerian element α_i.

Since the integrals of Eq. (19) cannot be solved in closed form, a Monte Carlo integration is adopted. The number of samples upon which the density is averaged is defined on the basis of the local value of the density gradient; indeed, the bigger is the variation of the density through the bin, the higher should be the number of samples to be taken to accurately estimate the density mean over the bin volume. The Monte Carlo integration integration is carried out in each bin of the domain, after having checked the fulfilment of three additional constraints:

1. There exists at least one set of Keplerian elements belonging to the bin such that the perigee is above the re-entry altitude.

2. There exists at least one set of Keplerian elements belonging to the bin whose related orbit intersects the parent object orbit in the fragmentation point.
3. The ejection velocity needed to reach the bin satisfies Eq. (5).

After the averaging procedure, each bin belonging to the domain probabilistically and physically reachable by fragments has an associated density value. The total number of fragments associated to the estimated density distribution can be easily computed as:

$$\widetilde{N} = V_\alpha \ \delta A/M \sum_i \overline{n_i} \tag{22}$$

3. Fragments' cloud density propagation and interpolation

The fragments' cloud density is propagated applying the method of characteristics [9] to the continuity equation, here recalled:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{F}) = 0 \tag{23}$$

where t is time, n the phase space density, $\boldsymbol{x}$ the phase space variables and $F = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}$ the dynamics. The characteristics are here propagated semi-analytically using the Planetary Orbital Dynamics (PlanODyn) suite [10], following the approach firstly introduced in [11]. The considered phase space variables are semi-major axis a, eccentricity e, inclination i, right ascension of ascending nodes Ω, argument of periapsis ω e area-to-mass ratio A/M. The characteristics to be propagated are randomly sampled from the bins; if the grid used for the estimation of the density is fine enough, one sample per bin is sufficient to describe the dynamical evolution of the cloud. On the contrary, if the fragments spread out in a huge domain, the steps computed according to the density gradient may be too big to grasp the dynamical behaviour of the fragments under orbital perturbations. Therefore, the number of characteristics to be extracted from each bin is defined as follows.

$$N_c = \prod_{j=1}^{4} \max\left(\frac{\delta x_j}{\delta x_j^{dyn}}, 1\right) \tag{24}$$

where N_c is the number of samples per bin; δx_j are the step-sizes in semi-major axis, eccentricity, inclination and area-to-mass ratio computed when estimating the initial density distribution and δx_j^{dyn} is the needed resolution in the same variables in order to estimate accurately the dynamical evolution of the fragments' cloud.

The coordinates of the sampled characteristics, which are at this point defined in the subset of Keplerian elements (a, e and i), are expanded in full set of coordinates, by imposing the condition of intersection with the parent object orbit in the fragmentation point. By imposing this constraint, four possible solutions are generated for each subset of Keplerian elements, each of them associated to a different density value, depending on the needed ejection velocity.

If propagated to the same epoch, the characteristics form a scattered point cloud in the phase space domain. Therefore, the density distribution has to be recovered through interpolation. The sparse matrix approach for binning proposed in [12] is here applied. Note also that a nearest neighbor-like interpolation among neighbouring bins is additionally implemented to avoid holes in the distributions.

4. Case study

The studied case takes inspiration from the real fragmentation of a Russian Ullage rocket motor, exploded in space at epoch 01/06/2016. The initial Keplerian elements are listed in Tab. 1.

Table 1 – Initial Keplerian elements. Ullage rocket motor fragmentation.

a [km]	e [-]	i [deg]	Ω [deg]	ω [deg]	f [deg]
16143.0	0.568	65.07	94.03	134.02	216.50

Fragments down to 1 cm are considered; adopting a scaling factor $S = 0.1$ [8], the NASA SBM estimates 950 fragments generated by the explosion of the motor. For computing the initial density distribution, the parameters ξ and $k(\eta)$ of Eqs. (3) and (4) are set equal to 0.997 and 2, which should allow to represent 97.4% of the distribution. The corresponding threshold values of area-to-mass ratio A/M and ejection velocity Δv are $6.604 \ \mathrm{m^2/kg}$ and $651.6 \ \mathrm{m/s}$. For the presented simulation, 10 bins in area-to-mass ratio are considered.
The initial density distribution is depicted in Figs. 1 and 2, separating between independent (a, e, i and A/M) and dependent (Ω and ω) phase space variables. The step-sizes computed according to the density gradient are reported in Tab. 2.

Table 2 – Step-sizes for the estimation of the initial density. Ullage rocket motor fragmentation.

δa [km]	δe [-]	δi [deg]
130.1	0.0057	0.69

Two characteristics are sampled from each bin, which lead to a total of 19802 characteristics to be propagated. The propagation is carried out in a simplified dynamical model, which considers the drag effect and the J_2 perturbation only, for a period of 20 years. The proposed model is validated against Monte Carlo propagation of the 950 fragments generated by the explosion. The comparison is done on the number of fragments in orbit over time. The results are presented in Fig. 3.
As it can be noticed, the initial number of fragments is smaller than the total, since approximately 50 fragments reentered in the atmosphere during the first revolution around the Earth. The two profiles follow almost the same trend and the difference between them oscillates between 3% and 5% of fragments with respect to the total. This means that the density distribution captures from 95% to 97% of fragments during the time considered. Furthermore, the profile in $\Delta N\%$ does not seem to follow a monotonic trend towards increasing error, which should ensure the validity of the model also over longer periods.

5. Conclusions

The main goal of this paper was to define a method for the propagation and interpolation of the fragments' cloud density able to deal with any orbital regime. The proposed approach probabilistically computes the initial phase space domain in semi-major axis, eccentricity, inclination and area-to-mass ratio according to the cumulative density functions in ejection velocity and area-to-mass ratio. To avoid unnecessary large domains, an averaging procedure through integration over the ejection velocity angles is efficiently implemented, which allows to increase the computational efficiency. Furthermore, the singularity in the semi-major axis variation is solved, allowing to extend the analysis to fragmentations with ejection velocities that cause some fragments to be injected on hyperbolic trajectories. The initial density distribution is estimated through Monte Carlo integration at bin level. In this framework, particularly relevant is the proposed autonomous definition of the step-size according to the gradient of the density with respect to the Keplerian elements. The validation analysis was still limited to a simplified dynamical model, where only the drag effect and the J_2 perturbations were considered. Since the method demonstrated to be accurate compared to the Monte Carlo simulation, future works will be devoted to apply the proposed model to more complex dynamical models and in different orbital regions.

6. Acknowledgements

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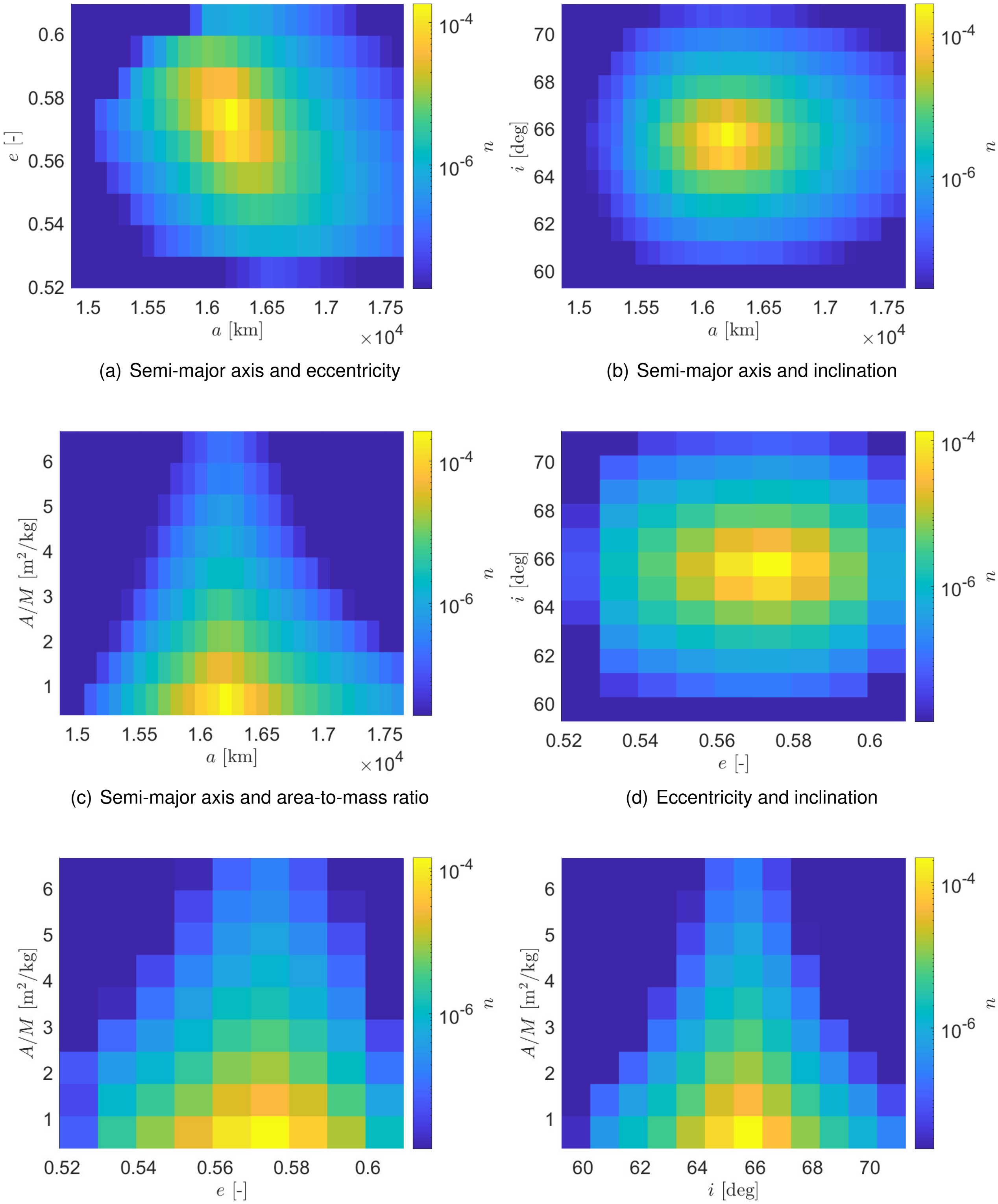


(a) Semi-major axis and eccentricity

(b) Semi-major axis and inclination

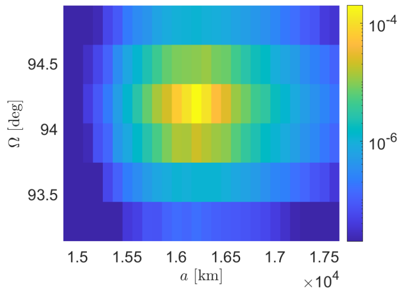
(c) Semi-major axis and area-to-mass ratio

(d) Eccentricity and inclination

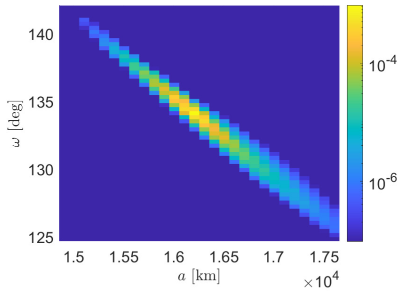
(e) Eccentricity and area-to-mass ratio

(f) Inclination and area-to-mass ratio

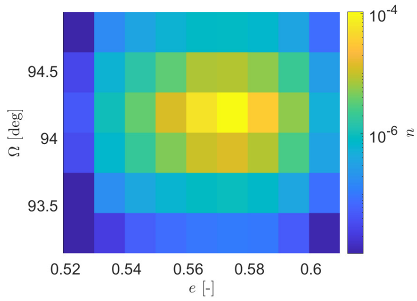
Figure 1 – Initial density distribution in semi-major axis, eccentricity, inclination and area-to-mass ratio. Ullage rocket motor fragmenation.



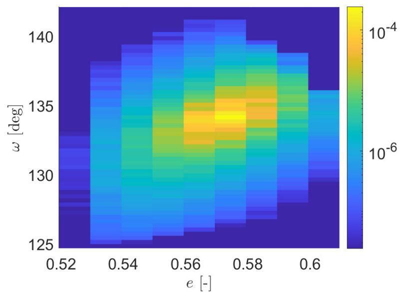
(a) Semi-major axis and right ascension



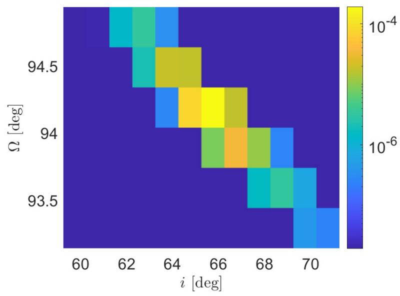
(b) Semi-major axis and argument of periapsis



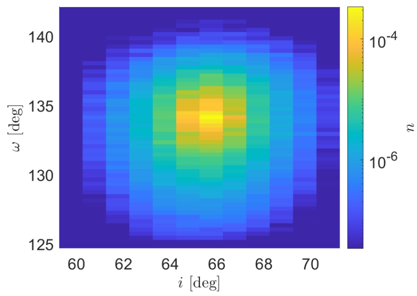
(c) Eccentricity and right ascension



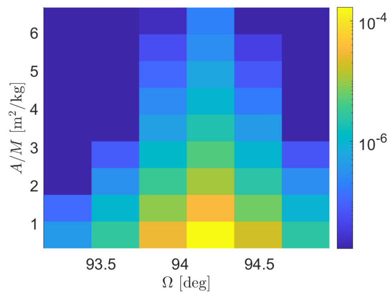
(d) Eccentricity and argument of periapsis



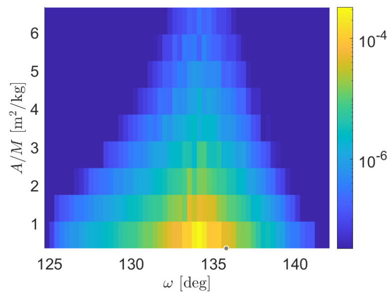
(e) Inclination and right ascension



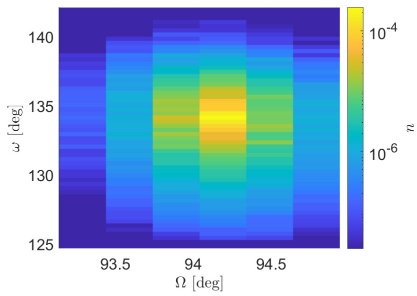
(f) Inclination and argument of periapsis



(g) Right ascension and area-to-mass ratio



(h) Argument of periapsis and area-to-mass ratio



(i) Right ascension and argument of periapsis

Figure 2 – Initial density distribution in right ascension of ascending nodes and argument of periapsis. Ullage rocket motor fragmenation.

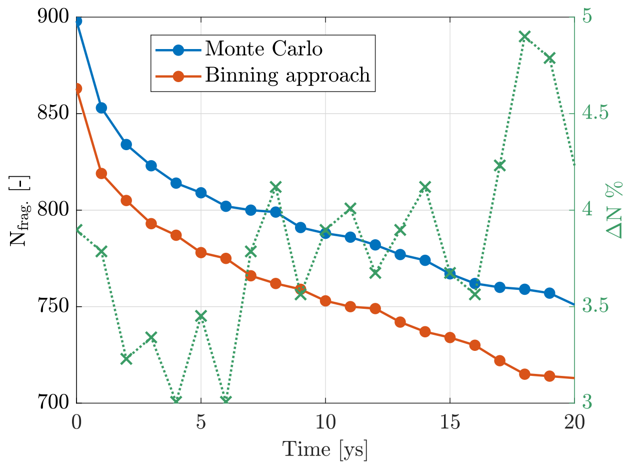


Figure 3 – Number of fragments over time, comparison between Monte Carlo and binning approach. Ullage rocket motor fragmenation.

7. Copyright Statement

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