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ANALYSIS OF LIGHTWEIGHT STRUCTURES USING PHYSICS INFORMED NEURAL NETWORKS

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Abstract

A novel machine learning method known as Physics-Informed Neural Netwworks (PINNs) is presented in this work for the analysis of lightweight structures. The training process is performed employing a recently developed learning algorithm referred as Extreme Learning Machine (ELM). This framework is first applied as a collocation method for solving numerically the linear static, buckling and free vibration problems of a composite plate with a hole. A second application is then presented for a wing box structure where the neural network is enriched with data from the system under analysis. The comparison of results against Finite Element (FE) simulations demonstrates the effectiveness of the proposed approach.

Keywords: Physics-Informed Neural Networks, Extreme Learning Machine, Lightweight Structures.

1. Introduction

The design process of aeronautical structures requires several analyses to be conducted to perform preliminary optimizations or parametric studies. Metamodelling techniques have been proposed as a viable mean to accomplish these tasks, while alleviating the whole computational burden [1]. An interesting example is given by the use of Artificial Neural Networks (ANNs) for designing structures operating in the nonlinear postbuckling regime, see [2]. When dealing with metamodels, one crucial aspect regards the availability of data for the training process. As a matter of fact, the time for generating training points via numerical simulations can be high, and this is even more amplified when they are produced via experiments. For this reason, any strategy which can reduce the amount of data required for training is of interest, while keeping in mind that any shortage of them may lead to poor generalization performances and learning difficulties of the metamodel. In this context, Physics-Informed Neural Networks (PINNs) [3] represent an emerging class of ANNs capable of integrating the often-limited training dataset with supplementary points, or collocation points, which bear the physics information of the problem under investigation and are not associated with any simulations or experiments. This additional knowledge is embedded in the loss function defined for the training process and can be in the form of any physical law or empirical relation.
The present work proposes a metamodelling technique based on PINNs combined with a novel learning algorithm, known as Extreme Learning Machine (ELM) [4], with application to the structural analysis of lightweight structures. The underlying physical laws governing the problem – in this case consisting of the equilibrium equations – are plugged into the loss function to enrich the information provided by the available training dataset. As compared with existing training strategies which rely on Gradient-based Learning (GBL) algorithms [5] and iteratively minimize the loss function for tuning the weights and biases of the network, ELM allows to perform this process in a single step by solving a least-square problem. This feature leads to improved training time, which can be several orders of magnitudes smaller with respect to the traditional approach.

2. Problem formulation

In this work, the physical information adopted for *informing* the PINNs are in the form of physical laws represented by partial differential equations (PDEs) and boundary conditions. These governing equations express the equilibrium conditions of an elastic plate and are derived in the context of the Kirchhoff thin plate theory [6]:

$$\begin{aligned} R_{\mathrm{x}}(\mathbf{u},\mathbf{x}) &= 0, \quad \mathbf{x} \in \Omega \\ B_{\mathrm{x}}(\mathbf{u},\mathbf{x}) &= 0, \quad \mathbf{x} \in \partial\Omega \end{aligned} \tag{1}$$

where R_{x} and B_{x} are two differential operators in $\mathbf{x} = [x, y]^{\mathrm{T}}$, while $\mathbf{u} = [u, v, w]^{\mathrm{T}}$ is the vector collecting the middle-plane displacement components of the plate along the two in-plane directions, x and y, and thickness direction, respectively.

The PINNs are applied here for solving three different class of problems, i.e. linear static, free vibration and buckling problems. Accordingly, the residual function R_{x} can be expressed as follow:

$$R_{\mathrm{x}}(\mathbf{u},\mathbf{x}) := K_{\mathrm{x}}(\mathbf{u},\mathbf{x}) - \beta_2\omega^2 M_{\mathrm{x}}(\mathbf{u},\mathbf{x}) + \beta_3\lambda G_{\mathrm{x}}(\mathbf{u},\mathbf{x}) + \beta_1\mathbf{q}(\mathbf{x}) = \mathbf{0} \tag{2}$$

where K_{x}, M_{x} and G_{x} are the differential operators defining the stiffness and mass properties, and the prebuckling state of the system, while ω is the vibration frequency, λ the buckling multiplier, and $\mathbf{q} = [q_x, q_y, q_z]^{\mathrm{T}}$ the vector of external loads. The quantities $[\beta_1, \beta_2, \beta_3]^{\mathrm{T}}$ are equal to 1 or 0 according to the problem of interest.

More complex structures can be obtained by assembling simple plate elements. In this case, the governing equations can be written as:

$$\begin{aligned} &\begin{cases} R_{\mathrm{x}}^{(p)}(\mathbf{u},\mathbf{x}) = 0 & \text{in } \Omega^{(p)} \\ B_{\mathrm{x}}^{(p)}(\mathbf{u},\mathbf{x}) = 0 & \text{in } \partial\Omega^{(p)} \end{cases} \quad \text{for } p = 1 \dots P \\ &\begin{cases} C_{\mathrm{x}}^{(q)}(\mathbf{u},\mathbf{x}) = 0 \\ E_{\mathrm{x}}^{(q)}(\mathbf{u},\mathbf{x}) = 0 \end{cases} \quad \text{in } \partial\Omega_{\mathrm{int}}^{(q)} \quad \text{for } q = 1 \dots Q \end{aligned} \tag{3}$$

where P is the total number of plate elements, Q the number of interfaces between elements, while $C_{\mathrm{x}}^{(q)}$ and $E_{\mathrm{x}}^{(q)}$ are differential operators which impose compatibility and the equilibrium conditions at the interfaces $\partial\Omega_{\mathrm{int}}^{(q)}$.

3. Methodology

3.1 Physics-Informed Neural Network

Artificial Neural Networks are commonly used as *black-box* approach, where the learning process is entrusted entirely on given sets of input/output data $[\mathbf{x}^i, \mathbf{u}^{i*}]$ which are extracted from the physical process under analysis. Due to the high cost of data acquisition in many engineering applications, training ANNs in a small data regime is a situation typically encounted in real-life problems. Often the generalization performances of the ANN are poor, and solutions can be not physical.

Physics-Informed Neural Networks are ANNs whose training process is enhanced and optimized with some prior knowledge of the problem under investigation [3]. This prior knowledge can be in the form of some generally accepted physical laws which are descriptive of the phenomenon under investigation and are not exploited in standard *black-box* ANNs. The training process of PINNs is performed by considering a loss function $\mathscr{L} = \mathscr{L}_{\mathrm{data}} + \mathscr{L}_{\mathrm{physics}}$ in which the information content of available input/output data is enriched with the physical understanding of the problem.

The data-driven part of the loss function can be defined as the mean square error:

$$\mathscr{L}_{\mathrm{data}} = \sum_{i=1}^{N_{\mathrm{data}}} \frac{|\mathbf{u}^i - \mathbf{u}^{i*}|^2}{2N_{\mathrm{data}}} \tag{4}$$

where N_{data} is the number of available data, $\mathbf{u}^i$ is the network output prediction for the i-th input data $\mathbf{x}^i$, whereas $\mathbf{u}^{i*}$ is the corresponding desired output.

For the problems considered here, the physics-based contribution can be expressed referring to Eq. (3) as follow:

$$\mathscr{L}_{physics} = \sum_{p=1}^{P}\left[\sum_{i=1}^{N_{\mathrm{f}}^{(p)}} \frac{|R_{\mathrm{x}}^{(p)}(\mathbf{u}^i,\mathbf{x}^i)|^2}{2N_{\mathrm{f}}^{(p)}} + \sum_{j=1}^{N_{\mathrm{b}}^{(p)}} \frac{|B_{\mathrm{x}}^{(p)}(\mathbf{u}^i,\mathbf{x}^i)|^2}{2N_{\mathrm{b}}^{(p)}}\right] + \sum_{q=1}^{Q}\left[\sum_{k=1}^{N_{\mathrm{int}}^{(q)}} \frac{|C_{\mathrm{x}}^{(q)}(\mathbf{u}^k,\mathbf{x}^k)|^2}{2N_{\mathrm{int}}^{(q)}} + \sum_{k=1}^{N_{\mathrm{int}}^{(q)}} \frac{|E_{\mathrm{x}}^{(q)}(\mathbf{u}^k,\mathbf{x}^k)|^2}{2N_{\mathrm{int}}^{(q)}}\right] \quad (5)$$

with N_{f}, N_{b} and N_{int} specifying the number of additional points, i.e. collocation points, which impose the equations in the domain, at the boundaries and interfaces, respectively.

3.2 Training via Extreme Learning Machine

The training process is carried out through the minimization of the cost function $\mathscr{L}$ in terms of the internal parameters of the network, i.e. weights and biases. Extreme Learning Machine (ELM) is a fast learning algorithm for training ANNs [4] which can be used as an alternative to Gradient-Based Learning (GBL) approaches [5].
Considering a single-hidden-layer neural network, its output is represented by:

$$u^i = \sum_{n=1}^{N_{\mathrm{n}}} c_n \sigma(\mathbf{w}_n \mathbf{x}^i + b_n) \quad (6)$$

where N_{n} the number of hidden neurons, c_n are the output weights, σ is the activation function for the hidden layer, $\mathbf{w}_n$ is the vector of internal weights connecting the inputs with the n-th hidden neuron, b_n is the corresponding bias.
The ELM trains only the outer layer weights whereas the other internal parameters of the network are chosen randomly. The training process is then performed in a single step through the solution of a least-square problem in the form of:

$$\mathbf{Lc} = \mathbf{t} \quad (7)$$

where $\mathbf{c}$ is the vector collecting the output weights, while $\mathbf{L}$ and $\mathbf{t}$ are obtained upon substitution of Eq. (6) in the expression of $\mathscr{L}$ and imposing the stationary condition, i.e. $\partial\mathscr{L}/\partial c_n = 0$ for $n = 1,...N_{\mathrm{n}}$. As opposed to GBL approaches, extremely high learning speed can be achieved due to the absence of any iterative process. These gains in computational time are obtained without affecting the generalization performances of the network [4].

4. RESULTS

In this section, the PINN-ELM framework is applied for the analysis of lightweight structures. Two examples are illustrated. Firstly, PINNs are adopted as a collocation method for the numerical solution of the PDEs given in Eq. (1). Secondly, they are presented as a hybrid metamodeling technique which makes use of both raw input/output data and physical knowledge of the problem.
In the first application, the loss function considered is $\mathscr{L} = \mathscr{L}_{\mathrm{physics}}$. In this case, the training process is solely based on physical laws which are assumed to be exact and fully representative of the physical process under analysis. For this reason, the PINN can be seen as a *white-box* approach which is in contrast with the traditional *black-box* use of ANNs for which $\mathscr{L} = \mathscr{L}_{\mathrm{data}}$.
In the second example, the training process is carried out based on the following loss function $\mathscr{L} = \mathscr{L}_{\mathrm{data}} + \mathscr{L}_{\mathrm{physics}}$. In this latter case, the physical laws are assumed to be partially known, or available in an approximate way, and are used in conjunction with some data extracted from the system itself. Due to the heterogenity of information provided during the learning process, the PINN can be viewed now as a *grey-box* approach.

4.1 Flat plate with cutout

As first illustrative example, a flat composite plate with cutout is considered. The planar dimentions are taken as $a = 200$ mm and $b = 100$ mm, while the radius of the circular cutout is $r = 25$ mm. The material properties are: $E_{11} = 138000$ MPa, $E_{22} = 8960$ MPa, $G_{12} = 7100$ MPa, $\nu_{12} = 0.3$ and $\rho = 1.5\times10^{-9}$ kg/mm^3. Two different lamination sequences are considered: a quasi-isotropic laminate $[\pm45/0/90]_{2s}$ and a cross-ply laminate $[0/90/0/90]_{2s}$. The ply thickness is $t = 0.1$ mm.

Three different types of analysis are carried out: linear static, buckling and free vibration analysis. For the linear static analysis, a uniform compressive load $\overline{N}_{xx} = 1$ N/mm is applied at the short edges of the plate. Due to the symmetry of the problem, only a quarter of the plate is analyzed. For the other two problems, the four edges of the quarter-plate are simply supported.
The neural networks share the same the shallow architecture illustrated in Figure 1. In all cases, approximately $N_{\text{physics}} = N_{\text{f}} + N_{\text{b}} = 1000$ collocation points and $N_{\text{n}} = 600$ hidden neurons are considered for the training procedure, while hyperbolic tangent is adopted as the activation function in the hidden layer. The results are compared with Finite Element (FE) simulations and are summarized

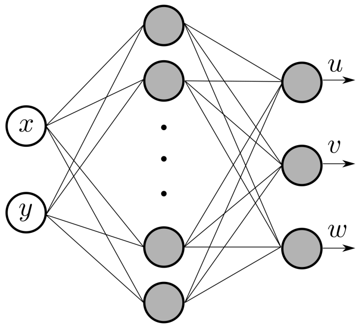

Figure 1 – Neural Network architecture.

in Table 1 in terms of maximum force resultant $N_{xx}^{\max}$, critical buckling coefficient λ_1 and fundamental natural frequency f_1. Close agreement is observed between PINN and FE method with maximum percentage errors of $E_{\%}[N_{xx}^{\max}] = 3.92\%$, $E_{\%}[\lambda_1] = 5.12\%$ and $E_{\%}[f_1] = 2.50\%$. To give further proof of

	$N_{xx}^{\max}$ [N/mm]			λ_1 [-]			f_1 [Hz]		
	PINN	FE	$E_{\%}$	PINN	FE	$E_{\%}$	PINN	FE	$E_{\%}$
$[\pm 45/0/90]_{2s}$	-4.24	-4.08	3.92	184.4	184.7	0.16	2119.0	2102.5	1.34
$[0/90/0/90]_{2s}$	-5.13	-5.30	3.20	164.0	156.0	5.12	2040.8	1991.0	2.50

Table 1 – Comparison between PINN and FE method for the linear static, buckling and free vibration solutions.

the effectiveness of the present method as PDEs solver, the contour plots of the resultant N_{xx}, the first buckling and vibration modes are presented in Figure 2. Even in this case, the accuracy of PINN results can be noted by comparison with the FE method. Interesting to note that the computational time of a PINN/ELM analysis is similar to a FE simulation. This example demonstrates the potentiality of using PINN and ELM for accurate and fast preliminary structural analysis alternative to other numerical methods.

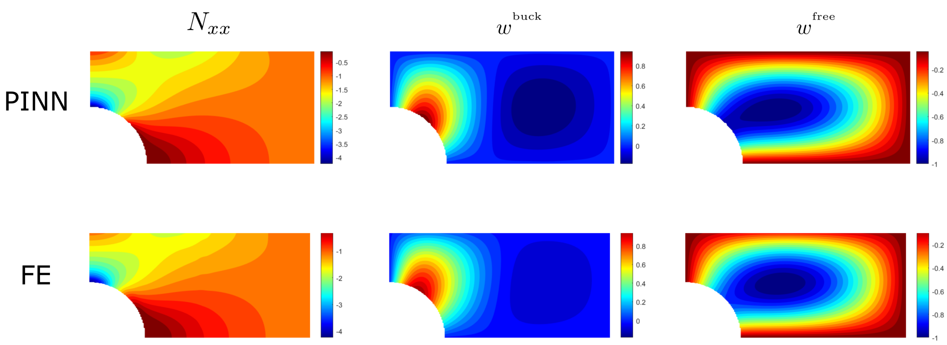

Figure 2 – Comparison between PINN and FE method for the linear static, buckling and free vibration solutions for lamination sequence $[\pm 45/0/90]_{2s}$.

4.2 Wing box

The second illustrative example regards a wing box structure whose geometry is outlined in Figure 3. Skins, spar webs and stringers are assumed to have the same thickness $t = 1$ mm and are made of an light aluminum alloy whose properties are: $E = 73000$ MPa and $\nu = 0.33$. The wing box is clamped at one end and free at the other. The free end is stiffened by a rib, which is used for load introduction. The PINN approach is now used as an hybrid metamodeling technique, i.e. *grey-box*, for predicting

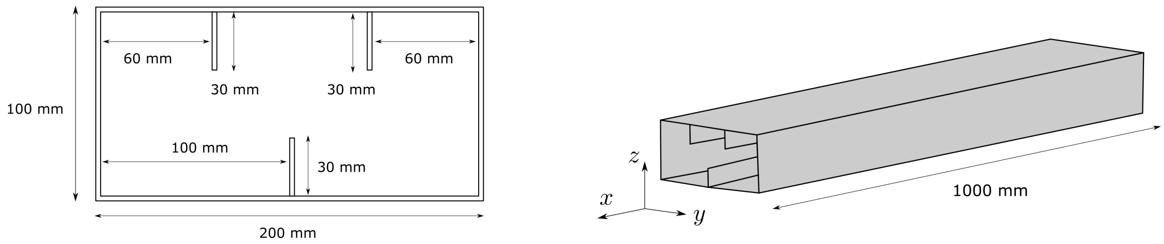

Figure 3 – Geometry of the wing box.

the linear static response under the effect of a concentrated vertical force F acting at the free end – the force is applied at the shear center of the section in order to avoid any torsional effects and with a magnitude such to have a vertial rigid motion of the rib section equal to $\overline{w} = 1$ mm.

The wing box of Figure 3 is modeled as an assembly of 10 plate elements: three and two for the upper and lower skin, respectively, and one for each spar and stringer. These plate elements are linked to each other through 7 interfaces. Due to the topology of the structure, a system of 10 PINNs is considered for modeling the whole structure, i.e. one for each plate element. Each PINN has the same shallow architecture of Figure 1 with a number of hidden neurons of $N_n = 100$ and the hyperbolic tangent as activation function. The set of training data accounts for $N_{\text{physics}} = 7500$ collocation points, approximately, and $N_{\text{data}} = 30$ input/output data points. These latter can be understood as measurements available from an external source and are artifically generated here from the FE model of the structure. In particular, they are generated performing a linear interpolation of the FE solution at the node points – a sketch of the training data distribution is reported in Figure 4.

The deflected shape predicted by the PINNs after training is shown in Figure 5 along with the one

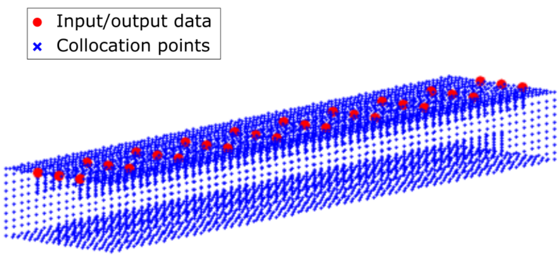

Figure 4 – Training data distribution in the wing box.

provided by FE simulations – an amplitude factor of 100 has been used for generating the figures. As seen, the PINNs are leading to results very similar to those obtained with the FE method, despite the network is feeded with few data points. To further assess the quality of PINN predictions the distribution of internal force resultants, N_{xx}, N_{yy} and N_{xy}, are presented in Figure 6 for a portion of the lower skin. Again, an excellent degree of accuracy is demonstrated.

Despite the more complex neural network architecture and the relatively large number of collocation points, the time for training is of the order of few tens of seconds. On the contrary, a standard GBL approach would require a total time of one or two orders of magnitude larger. This example provides a clear proof of the beneficial effect given by the introduction in the learning process of the physical understanding of the problem which permits to amplify the information content of the available data. Indeed, if traditional *black-box* ANNs were employed with the same number of input/output data it would have been impossible to extract the same amount of information from the system, i.e. the complete structural response of the wing box.

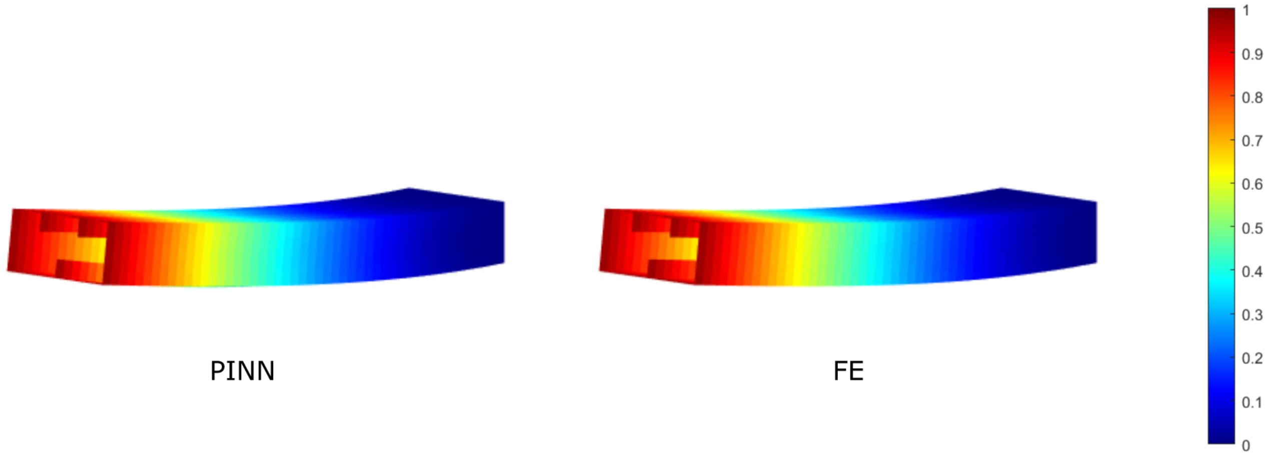

Figure 5 – Comparison of the static deflection shape of the wing box between PINN and FE method.

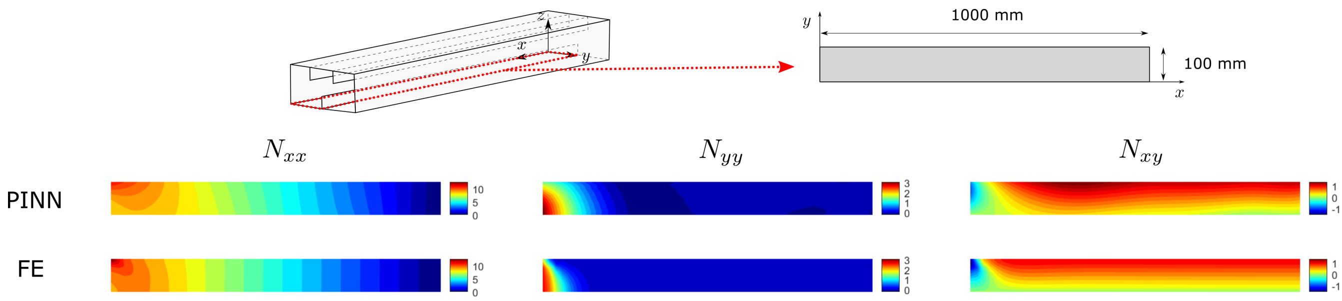

Figure 6 – Comparison of internal force resultants of a portion of the wing box between PINN and FE method.

5. CONCLUSIONS

In the present work, a data-efficient and computationally rapid metamodeling technique has been presented for the analysis and modeling of lightweight structures. This framework is based on the combination of two novel machine learning methods, i.e. Physics-Informed Neural Networks and Extreme Learning Machine. The introduction of some prior knowledge of the problem in the learning process of neural networks allows to achieve solutions which are physically consistent with the actual behavior of the system, even in a low data regime or in total absence of them. Additionally, the adoption of ELM for training PINNs gives a boost to the learning procedure with drastic reduction of training times compared to traditional learning strategies based on the gradient. The present framework has been applied in two different ways: as a *white-box* approach for the solution of specific PDEs, and as a *grey-box* approach for the construction of hybrid metamodels based on raw input/output data and physics information. Comparison with FE simutations provides evidence of the validity of the proposed methodology. For this reason, the present framework has the potential to be applied in a wide range of problems arising in the aerospace sector, including structural analysis, design optimization, surrogate modeling and model parameters identification.

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7. Copyright Statement

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