Space debris cloud propagation through phase space domain binning

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It is estimated that almost one million debris greater than 1 cm currently orbit the Earth, posing hazard to operational satellites. The traditional piece-by-piece approach, to monitor the evolution of such small space debris, is computationally heavy. This problem has been elegantly addressed through an analogy with fluid dynamics. In [1], [2] the method of characteristics (MOC) was adopted to find analytical solutions to the continuity equation in the phase space of a subset of Keplerian elements, under simplified orbital regimes, which limited the analysis to fragmentations in low-Earth orbits. The Starling suite was developed at Politecnico di Milano [3] in the framework of the continuum approach, to deal with debris fragments propagation of any dimension and subjected to non-linear dynamics. This approach allowed to potentially extend the continuum approach to any orbital regime. In [3], a Gaussian mixture model was selected as density interpolation technique; however, currently this method cannot account for forces that lead to resonances on a small subset of the phase space, as it could be the case of third-body perturbation or solar radiation pressure.

This paper aims at developing a method for propagation and interpolation of the fragments' cloud density able to deal with any orbital regime. The goal is achieved combining the MOC, applied to the continuity equation, with the phase space splitting into bins. The initial fragments' density distribution, associated to a fragmentation event, is computed through an analysis on the likelihood of a fragment to reach a certain region (i.e., bin) of the phase space of Keplerian elements and area-to-mass ratio. This is done mapping a 4D distribution in cartesian velocity and area-to-mass into a 7D distribution in Keplerian elements and area-to-mass [4], in the domain probabilistically reachable by the ejected fragments. A set of representative samples, selected from the bulk of non-empty bins at fragmentation epoch, is numerically propagated through the MOC. The forcing term is modelled to suitably represent the orbital regime under which the fragments' cloud evolves; furthermore, marginalisation and, hence, dimensionality reduction of the problem over some angular phase space variables is accounted, if the fragments are approximately uniformly distributed in the related direction. The density distribution is recovered at some reference times by averaging the density values of the propagated samples that share the same bin; interpolation among bins occupying neighbouring areas is considered to ensure a smoother, and more physically consistent, variation of the density in time. Since the debris generated by a fragmentation event remain bounded within certain regions of the phase space, the issue of storing such heavy multi-dimensional distributions is faced through well-proven tools for sparse matrices storage [5].

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