## Analysis of Lightweight Structures using Physics Informed Neural Networks

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The design process of aeronautical structures requires several analyses to be conducted to perform preliminary optimizations or parametric studies. Metamodelling techniques have been proposed as a viable mean to accomplish these tasks, while alleviating the whole computational burden. An interesting example is given by the use of Artificial Neural Networks (ANNs) for designing structures operating in the nonlinear postbuckling regime, see [1]. When dealing with metamodels, one crucial aspect regards the availability of data for the training process. As a matter of fact, the time for generating training points via numerical simulations can be high, and this is even more amplified when they are produced via experiments. For this reason, any strategy which can reduce the amount of data required for training is of interest, while keeping in mind that any shortage of them may lead to poor generalization performances and learning difficulties of the metamodel. In this context, Physics-Informed Neural Networks (PINNs) [2] represent an emerging class of ANNs capable of integrating the often-limited training dataset with supplementary points - i.e. collocation points which are not associated to any simulations or experiments - which bear the physics information of the problem under investigation. This additional knowledge is embedded in the loss function defined for the training process and can be in the form of any physical law or empirical relation.

The present work proposes a metamodelling technique based on PINNs combined with a novel learning algorithm, known as Extreme Learning Machine (ELM) [3], with application to the structural analysis of lightweight structures. The underlying physical laws governing the problem – in this case consisting of the equilibrium equations – are plugged into the loss function to enrich the information provided by the available training dataset. As compared with existing training strategies which rely on Gradient-based Learning (GBL) algorithms [4] and that iteratively minimize the loss function for tuning the weights and biases of the network, ELM allows to perform this process in a single step by solving a least-square problem. This feature leads to improved training time, which can be several orders of magnitudes smaller with respect to the traditional approach.

An example showing a potential application of the proposed PINN approach is provided in Figs. 1-4. The problem consists in obtaining the internal membrane resultant  $N_{xx}$  of a panel with a cutout and loaded with a tensile force at its end. The results from Finite Element (FE) analysis are used as reference and for simulating the available training dataset, here constituted by the FE solution evaluated at the nodal points. The solutions of two different configurations of PINN are compared against the one of a standard ANN trained with a GBL algorithm, see Fig. 2. Specifically, the results available from a PINN trained using ELM in complete and in partial absense of training points are reported in Figs. 3-4, respectively. The comparison between the contours clearly demonstrates the beneficial effects of PINN in obtaining more regular solutions while requiring less training data. At the same time, drastic saving is achieved in terms of CPU time due to the adoption of ELM.

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Fig. 1 Finite Element solution used for reference and for generating training points.



Fig. 2 Standard ANN trained with GBL using the training points taken from the FE solution.



Fig. 3 PINN trained with ELM using only collocation points - completely absense of training points.



Fig. 4 PINN trained with ELM using collocation points and a limited set of training points.

## References

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